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THE FRACTIONAL-ORDER PID CONTROLLER OUTPERFORMS THE CLASSICAL ONE

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Abstract: The three-parameter PID controller is the most popular industrial controller thanks to its simplicity. Only two more parameters arise after considering the integrator and derivator of arbitrary real order. Such a generalized controller is called fractional-order PID controller (FPID). Moreover, these two parameters (the order of integrator and derivator) still have a clear physical interpretation. This paper extends the results of the author's previous papers where the robustness regions method for traditional PID controller was discussed. The aim of this paper is to present a generalized robustness regions method for the FPID controller. The proposed procedure allows to define several frequency domain closed loop requirements simultaneously - gain and phase margins, limitations of sensitivity functions, proper bandwidth. The final example shows that the FPID controller ensures the fulfillment of stricter closed-loop requirements than the traditional PID. The authors believe that the FPID controller could be a suitable candidate for some specific industrial applications.

Keywords: fractional-order controller, PID, robustness regions, Nyquist plot shaping

1 INTRODUCTION

Over the years, the PID controller becomes one of the most popular industrial controllers thanks to its simplicity and the ability to tune a few parameters automatically. The boom of the fractional calculus in the last decades showed new possibilities for process identification (Charef and Sun [1992], Čech and Schlegel [2006]) and controller synthesis (Podlubny [1999]). In this area, the fractional-order PID controller (FPID) arises from the classical one after considering integrator and derivator of arbitrary real order. Moreover, these two parameters still have a clear physical interpretation. The generalized fractional order operators are also linear, so the same methods and rules as for the frequency domain linear control system design can be applied.

In linear system theory, the overall quality of the control loop can be evaluated especially in the frequency domain - gain and phase margins, limitations on sensitivity functions, proper bandwidth, low frequency disturbance dumping, etc. To proceed with these design specifications, the robustness regions method for a classical PID controller with filtered derivative part was presented in the author's previous work Schlegel and Čech [2004b]. It is based on classical D-partition method (Shafei and Shenton [1997]). This method allows robust design for several process models as well. Hovewer, it may be difficult to fulfill all design specifications because of lower flexibility of the classical PID controller.

The aim of this paper is to present the generalized robustness regions method for a FPID controller with filtered derivative part. It will be shown that the FPID controller as a natural generalization of the classical one can fulfill stricter contradictory design specifications simultaneously. Note that in this paper, all aspects and features of FPID controller will be outlined only in frequency domain.

The paper is organized as follows: Section 2 introduces fractional order systems and the FPID controller. Basic facts about robustness regions method are reminded in Section 3. Also the main result of the paper - the generalized robustness regions method for FPID controller can be found in Section 3. Illustrative example is shown in Section 4. Concluding remarks and ideas for further work can be found in Section 5.

2 FRACTIONAL-ORDER SYSTEMS AND FPID CONTROLLER

Fractional calculus is usefull in both process identification and controller synthesis areas. From the identification point of view, fractional-order transfer functions enable to express arbitraty fractional

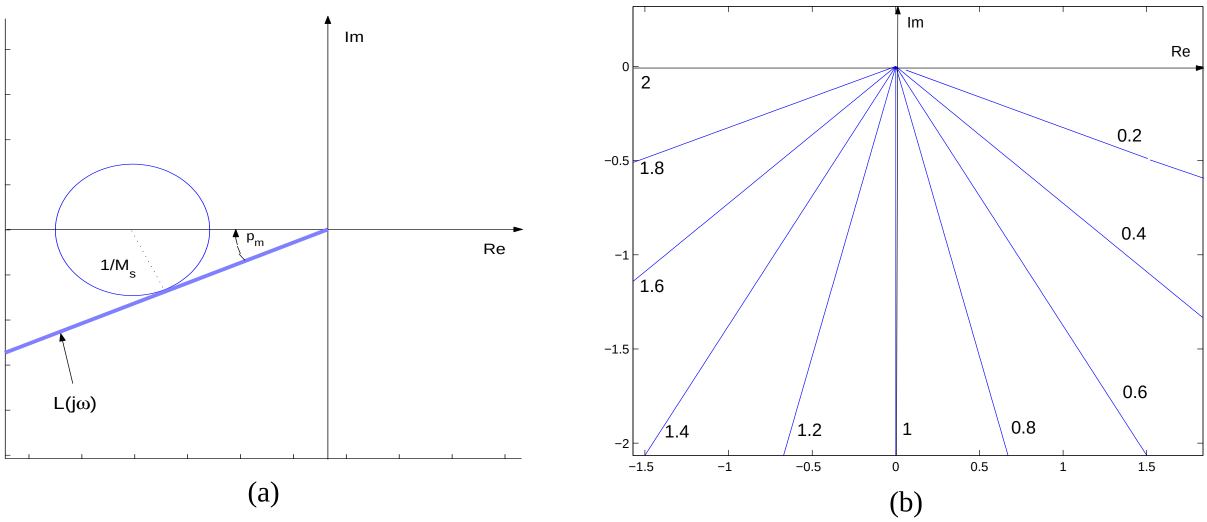


Figure 1 – (a) Ideal Nyquist plot shape, (b) Frequency responses of the fractional-order integrator for different α.

slopes of Bode plots of real processes (temperature, chemical) (Charef and Sun [1992]). Using simple fractional operators in the controller synthesis, one has wider Nyquist plot shaping possibilities.

2.1 Basic fractional-order operators

Firstly, let us consider the basic fractional-order integrator described by the transfer function

$$F(s) = \frac{1}{s^{\alpha}}, \quad \alpha \in \mathrm{R}. \tag{1}$$

The frequency responses of such a fractional-order integrator for different orders α are shown in Fig. 1 a. It is evident, that after adding fractional-order integrator into the control loop the Nyquist plot can approximate to an ideal shape proposed by Bode (Fig. 1 b). Such an ideal shape represents an infinite gain margin while the phase margin and the sensitivity function upper limit are independent on the open loop gain.

Similarly, the fractional-order derivator can be represented by the transfer function

$$F(s) = s^{\beta}, \quad \beta \in \mathrm{R}. \tag{2}$$

The fractional-order operators are linear thus the frequency responses can be computed and drawn.

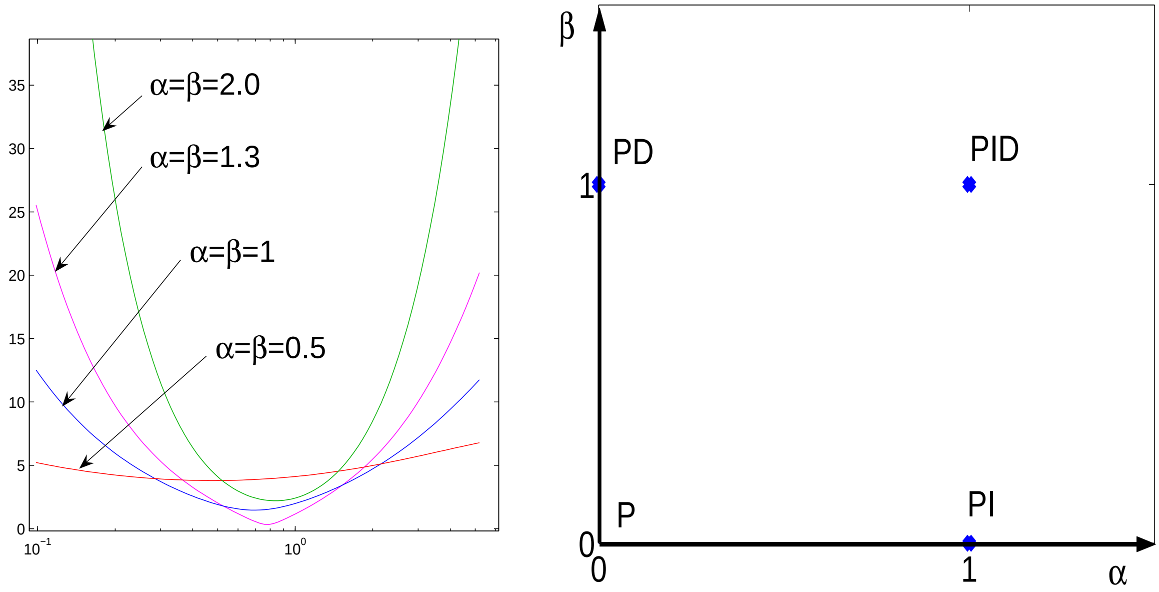
2.2 FPID controller

Over the years, the researchers as well as industrial practitioners aspired to substitute the traditional PID algorithm by some more powerful one. A lot of sophisticated optimal, adaptive and higher order control algorithms sprung up from this tendency. However, there were too complicated and not reliable. Hence the simple PID controller stays the most popular one in industrial sphere.

Substituting integer-order operators by fractional-order ones, the FPID controller with transfer function

$$C(s) = K\left(1 + \frac{1}{T_i s^{\alpha}} + T_d s^{\beta}\right) \tag{3}$$

grows up. The FPID controller as so as the classical PID behaves as a band stop filter. Changing integrator and derivator orders one can shape the filter sharpness independently as shown in Fig. 2a. Note that the FPID parametres α, β can be represented in the orders plane (Fig. 2b). In this Figure, also the special cases of traditional controllers are highlighted.



(a) (b)

Figure 2 – (a) FPID frequency response shaping, (b) FPID orders plane

For further practical digital realization, the derivative part has to be complemented by the first-order filter

$$C(s) = K\left(1 + \frac{1}{T_i s^\alpha} + \frac{T_d s^\beta}{\frac{T_d}{N}s + 1}\right). \tag{4}$$

This filter helps especially to dump high frequency noises in the control loop.

3 GENERALIZED ROBUSTNESS REGIONS METHOD

It is well known, that required loop performance (e. g. gain and phase margins) could be reached by shaping of the Nyquist plot $L(\mathrm{j}\omega) = C(\mathrm{j}\omega)P(\mathrm{j}\omega)$. Here, C denotes controller and P the process model. For example, the minimum gain margin 2 ($G_m \geq 2$) and minimum phase margin 60^o ($P_m \geq 60^o$) are equivalent with requirements that points $X_1 = -1/2$ and $X_2 = -1/2(1 + \mathrm{j}\sqrt{3})$ lie on the left side of $L(\mathrm{j}\omega)$ or just on this curve. If the specifications $G_m \leq 3$ and $P_m \leq 90^o$, the points $X_3 = -1/3$ and $X_4 = -\mathrm{j}$ should lie on the right of $L(\mathrm{j}\omega)$. In such a way the points X_1, X_2, X_3, X_4 define the required Nyquist plot shape.

Similarly, one can proceed with, if the limitation of the sensitivity function $S(\mathrm{j}\omega) = 1/(1 + L(\mathrm{j}\omega))$ is required in the form $|\sup(S(\mathrm{j}\omega))| < M_S$ or if the limitation of the complementary sensitivity function $T(\mathrm{j}\omega) = L(\mathrm{j}\omega)/(1 + L(\mathrm{j}\omega))$ is required in the form $|\sup(T(\mathrm{j}\omega))| < M_T$. It is well known that these requirements are equivalent with the requirement that the Nyquist plot does not have intersection with so called M-circles.

If we want to provide the specified disturbance rejection at a given frequency range and the proper bandwidth of the closed loop, the following conditions are suitable

$$\begin{aligned} |\sup(S(\mathrm{j}\omega))| &\leq \epsilon_S, \quad \omega \in <0, \omega_S> \\ |\sup(T(\mathrm{j}\omega))| &\leq \epsilon_T, \quad \omega \in <\omega_T, \infty> . \end{aligned} \tag{5}$$

3.1 One shaping point and FPI controller

Now, let us show the idea principle of robustness regions method in the case, when only one shaping point $X = u + \mathrm{j}v$ is specified in the Nyquist plot plane. Our aim is to find all possible pairs of

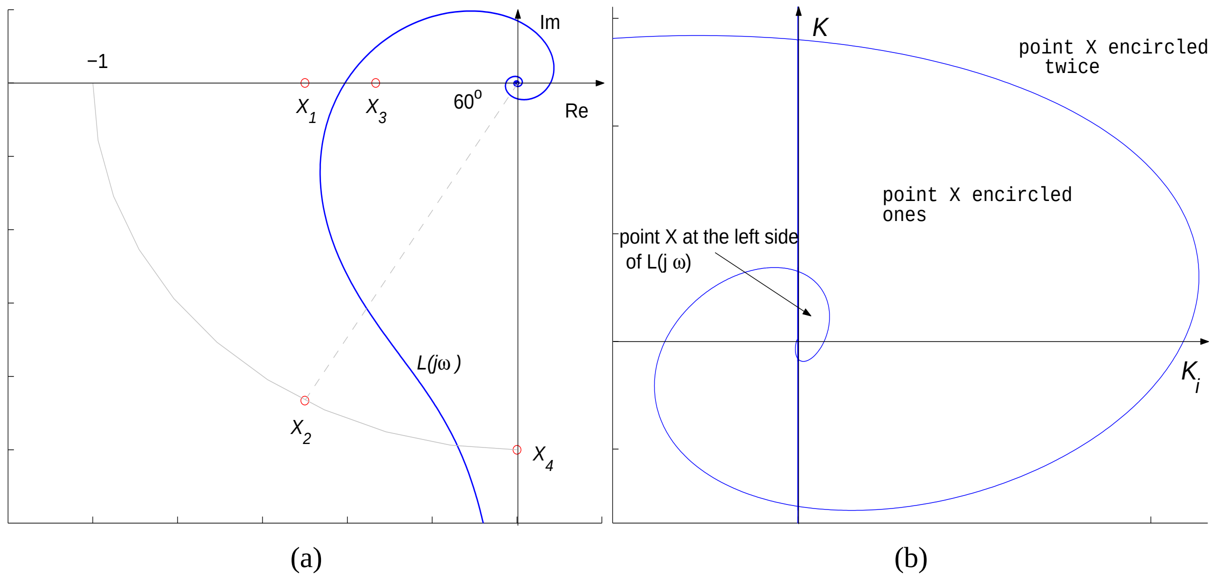


Figure 3 – (a) Nyquist plot shaping, (b) Robustness regions

parameters K, K_i of the FPI controller with transfer function

$$C(s) = K + \frac{K_i}{s^\alpha} \tag{6}$$

for which the point X lies on the left side of the Nyquist curve. For this purpose, let us solve the equation

$$L(\mathrm{j}\omega) = \left(K + \frac{K_i}{(j\omega)^\alpha}\right)(a(\omega) + \mathrm{j}b(\omega)) = u + \mathrm{j}v \tag{7}$$

for unknown K, K_i and fixed α, where $a(\omega) = \mathrm{Re}\{P(\mathrm{j}\omega)\}$ and $b(\omega) = \mathrm{Im}\{P(\mathrm{j}\omega)\}$. After some calculations one obtains

$$\begin{aligned} K(\omega) &= \frac{a(\omega)v\cos\phi + b(\omega)v\sin\phi + a(\omega)u\sin\phi - b(\omega)u\cos\phi}{(a^2(\omega) + b^2(\omega))\sin\phi} \\ K_i(\omega) &= \frac{(a(\omega)v - b(\omega)u)\omega^\alpha}{(a^2(\omega) + b^2(\omega))\sin\phi}, \end{aligned} \tag{8}$$

where $\phi = \frac{1}{2}\alpha\pi$. The expressions (8) define the parametric curve in the K, K_i plane. This curve together with K, K_i axis splits the parametric plane into several regions as it is shown in Fig. 3b. All points inside any region lead to the Nyquist plot having point X at the same side. More precisely, the number of encirclements of the point X by the Nyquist plot is the same. Usually just one region contains suitable points corresponding to the required relative location of the Nyquist plot and the point X. Using the same approach, we obtain also parametric curves in the case of SF and CSF design specifications (5).

General Nyquist plot shaping problem with a finite number of shaping points can be converted to the several one-point case described above. Here we have to find the intersection of all regions produced by individual shaping points. The same idea solves the problem of robust design for several process models.

3.2 FPID controller with filtered derivative part

If one wants to illustrate robustness regions for the FPID controller described by (4) in K, K_i, the four controller parameters α, β, N and $f = T_i/T_d$ have to be chosen *a priori*. Fortunately, parameter N has the clear physical interpretation. When $N \to \infty$, we obtain an ideal PID controller, while the derivative part is switched off if $N \to 0$. The value of N is usually chosen in the interval $\langle 1, 20 \rangle$ according to the signal/noise ratio. Moreover, the ratio $f = T_d/T_i$ is used to be equal to $1/4$ (Ziegler and Nichols [1942]). Newer studies (Äström and Hägglund [1995]) acknowledged correctness of this value especially for plants with a monotone step response. Note, that $f = 0$ leads to PI controller and $f > 1/4$ enhances the derivative part.

There are no general rules for the choice of integrator order α and derivator order β. The choice is dependent on the frequency domain closed loop requirements as it will be shown in the illustrative example. Reasonable values $\alpha, \beta \in \langle 0, 2 \rangle$.

Computation of region boundaries is much more complicated but the design technique can be used without changes. For each frequency ω, the roots of 3rd-order polynomial are computed. Therefore the region consists of several branches. The detailed results are ommited for brevity.

To make a design procedure routinely applicable, we decided to create an interactive Java applet described in Čech and Schlegel [2006].

4 ILLUSTRATIVE EXAMPLE

Let us assume that two extremal transfer functions were obtained in moment identification experiment (see Schlegel and Čech [2004a])

$$F_1(s) = \frac{1}{(0.6s+1)^{1.666}}, \tag{9}$$

$$F_2(s) = \frac{1}{(0.058s+1)^4(0.758s+1)}. \tag{10}$$

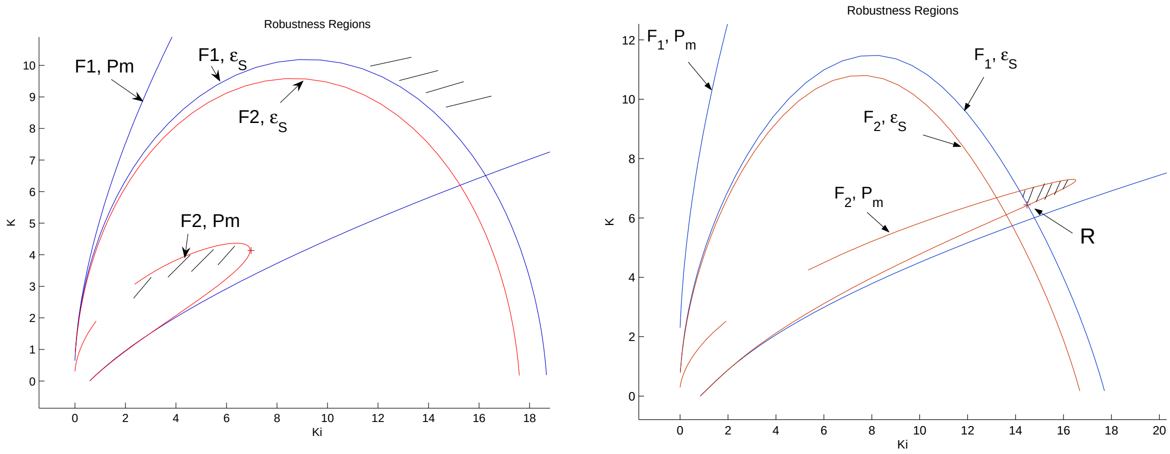
Further, the following design specifications are defined:

- $P_m > 60^o$ - phase margin
- $|M_S| < 2.0$ - sensitivity function upper limit
- $|M_T| < 1.3$ - complementary sensitivity function upper limit
- $\omega_S = 1.5[rad/s], \epsilon_S = 0.15$ - low frequency disturbance dumping - the sensitivity function should be less than ϵ_S till the frequency ω_S

It is impossible to fulfill these contradictory design specifications simultaneously using classical PID controller. For fixed values $N = 10, f = 0.25$, the intersection of admissible regions for P_M a (ω_S, ϵ_S) is empty, as shown in Fig. 4 a. The non-empty intersection can be reached only by increasing f. However, then the requirements on sensitivity functions (M_S, M_T) are never fulfilled. Choosing the FPID controller with integrator order $\alpha = 0.9$ and derivator order $\beta = 1.25$ leads to non-empty intersection and therefore the acceptable controller R exists (Fig. 4 b). The other controller paramaters are $K = 6.39,\ K_i = 14.53,\ f = 0.25,\ N = 10$. Resulted well shaped Nyquist plot are shown in Fig. 5. Here one can persuade, that also the sensitivity function requirements given by M-circles are fulfilled.

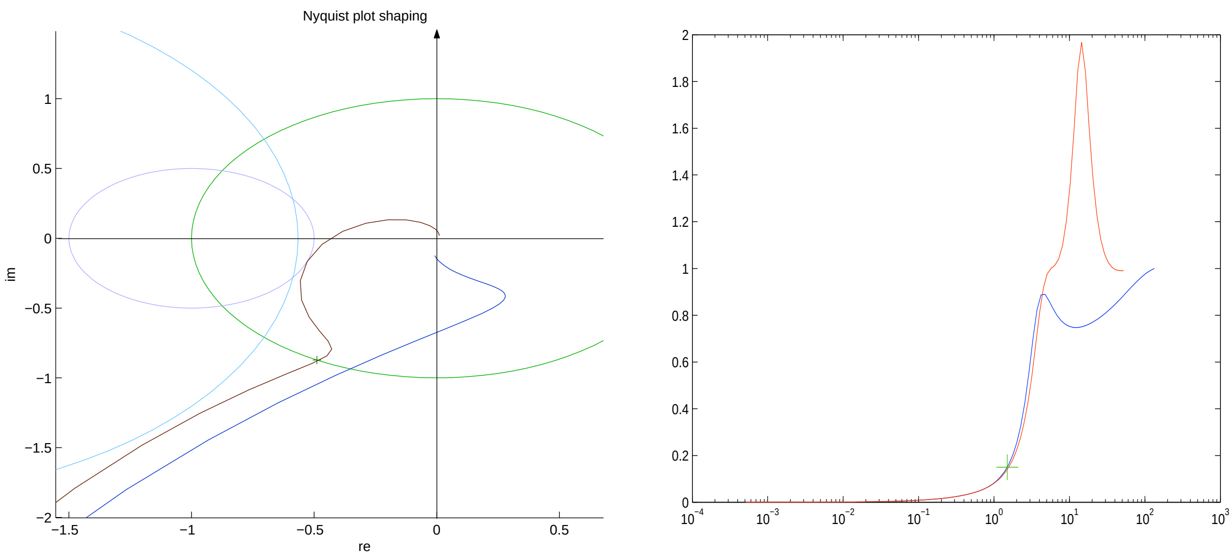
5 CONCLUSIONS

The aim of this paper was to present the generalized robustness regions method for FPID design. Using Nyquist plot shaping, this method allows to fulfill several design specifications simultaneously (gain and phase margins, sensitivity functions limitations, disturbance dumping, proper bandwidth). Sometimes it is impossible to ensure these contradictory requirements by a classical PID controller because of its lower flexibility. The illustrative example shows that the FPID controller can fulfill stricter contradictory design specifications simultaneously while the design procedure is still intuitive and the two additional parameters have a clear physical interpretation. In the future, we have to concentrate on digital time-domain realization of FPID controller for real-time industrial applications.



(a) (b)

Figure 4 – Robustness regions for a) PID controller, b) FPID controller.



(a) (b)

Figure 5 – Well shaped a) Nyquist plots, b) sensitivity functions

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