GENERALIZED ROBUSTNESS REGIONS FOR PID CONTROLLERS

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INTRODUCTION

- Several methods of PID controllers tuning are used in practice.
- In this paper, the systematic procedure based on the classical D-partition for construction of robustness regions in the parameter plane is presented when it is required that the open loop Nyquist plot does not have an intersection with a given general conic.
- The open loop transfer function conic constraints can be interpreted in terms of gain and phase margins and of sensitivity and complementary sensitivity functions.

GENERALIZED ROBUSTNESS REGIONS

- Consider the feedback control system with the PID controller C(s) and the process P(s)

$$C(s) = k\left(1 + \frac{1}{T_i s} + T_d s\right) = k + \frac{k_i}{s} + k_d s \ ,$$

$$P(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0} \ e^{-Ds} \ ,$$

where k, T_i, T_d are its engineering parameters and $k_i = k / T_i$, $k_d = k T_d$ are corresponding gains.

- We restrict the problem to the case of two design parameters k and d for which it holds

$$k_i = kd \ ,$$

$$k_d = k\frac{f}{d} = \frac{fk^2}{k_i} \ ,$$

where $d = 1/T_i$ and f is a fixed parameter with the meaning $f = T_d / T_i$.

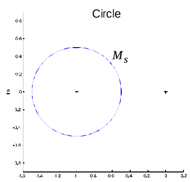
- We want to find all pairs (k, k_i) in the parameter plane for which the Nyquist plot

$$L(j\omega) = C(j\omega)P(j\omega) = u + jv$$

does not have an intersection with the conic V , but is tangential to it for some $\omega \in [0, +\infty)$. The conic is defined by the general equation

$$V: \ Kq^2 + Qr^2 + Mqr + Nq + Or + R = 0 \ .$$

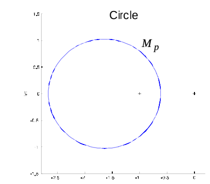
The general conic includes points, lines, circles, parabolas, ellipses and hyperbolas. Typical design specifications are given below.



$\sup_\omega |S(j\omega)| \le M_s$

$S(j\omega) = \dfrac{1}{1 + L(j\omega)}$

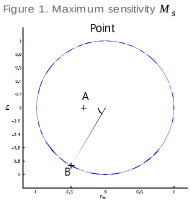
Figure 1. Maximum sensitivity M_s



$\sup_\omega |T(j\omega)| \le M_p$

$T(j\omega) = \dfrac{L(j\omega)}{1 + L(j\omega)}$

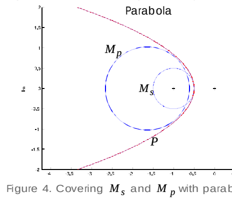
Figure 2. Maximum complementary sensitivity M_p



A: Gain margin G_M

B: Phase margin P_M

Figure 3. Gain and phase margins



$\sup_\omega |S(j\omega)| \le M_s$

$\sup_\omega |T(j\omega)| \le M_p$

Figure 4. Covering M_s and M_p with parabola P

- The main result is the effective algorithm for isolation of the robustness region for arbitrary conic constraint.

EXAMPLES

- *Example1.* Consider the following process

$$F(s) = \frac{(0.5s + 1)}{(0.25s + 1)^4} e^{-1.5s}$$

and the three constraints:
- the maximum sensitivity requirement $M_S = 2.0$,
- the phase margin 55 degrees,
- the parabola *P* with the top (0+ 0.4*j*) and focal distance *p* = -1.0.

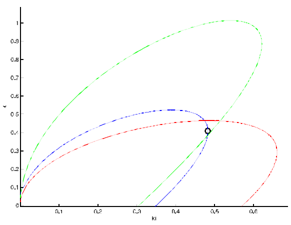


Figure 5. Robustness regions

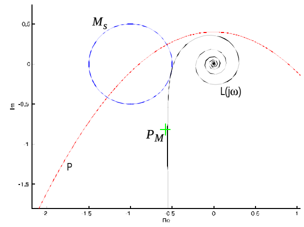


Figure 6. The Nyquist plot for ki=0.482; k=0.414.

- *Example2.* To illustrate the design of a robust PID controller, consider process models

$$F_1(s) = \frac{1}{(0.2s + 1)(0.4s + 1)^2} \ ,$$

$$F_2(s) = \frac{1}{(0.0864s + 1)^5 (0.5681s + 1)}$$

and the maximum sensitivity constraint $M_S = 2.0$.

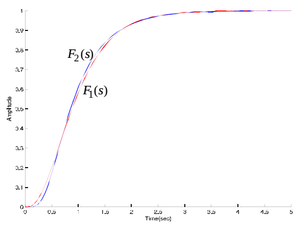


Figure 7. Step responses for $F_1(s)$ and $F_2(s)$

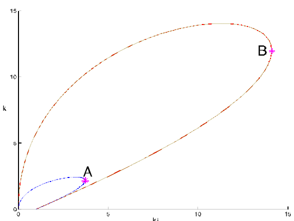


Figure 8. Robustness regions

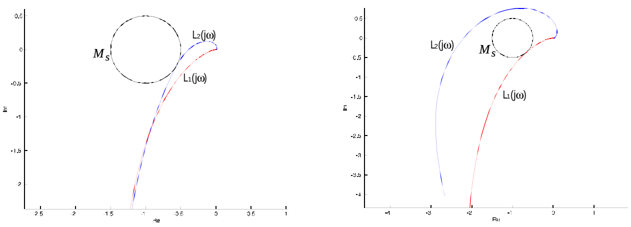


Figure 9. The Nyquist plots for (a) the point A; (b) the point B

- Remark:
  The controller designed for the process $F_1(s)$ gives the unstable loop with the process $F_2(s)$!

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