ARE HEURISTIC METHODS FOR PID DESIGN RELIABLE ?

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Abstract: Over the years, there are many heuristic formulas derived to design and tune the PID controllers. Most of them are developed and tested for first-order plus deadtime nominal system. The purpose of this paper is to evaluate some of these methods for the well defined class of systems including all common industrial processes.

Keywords: PID control, design, heuristic methods

1. INTRODUCTION

Over the years, there are many simple empirical method developed for PID tuning based mainly on the step response knowledge. The tuning formulas are not theoretically based, so we can't define the sphere of usage. These methods are often failing in practise, because of the difference between the real system and the testing system for formulas deriving (usually first order plus deadtime system). The popular $\kappa - \tau$ method has been tested on the set of processes, which looks very large, but does't cover all possible real processes.

The main purpose of this paper is to define the set of *a priori* admissible systems and test the existing methods on some extreme systems of this set. It is shown, that each method fails for some chosen admissible systems.

2. PROCESS MODEL SET

Usually used three characteristic numbers of the process correspond with infinite number of processes. The tuning formulas were optimized usually just for one process, so the reliability isn't well guaranteed. We have to consider all of the corresponding processes, but it isn't possible, of course. Firstly, we need to define the class of *a priori* **admissible systems**. Then the set of all corresponding systems will be well defined and can be explicitly specified.

Fig. 1. General model set creating schema

a priori admissible systems

Let's restrict on linear systems with transfer function in the form

$$F(p) = K\frac{\tau s + 1}{\sum\limits_{i=1}^{n}(\tau_i s + 1)}, \qquad (1)$$

where $\tau_i > 0,\ i = 1, \ldots, n,\ K > 0,\ \tau > 0$. All of this systems have monotonous step response and frequency response under the assumption that τ_0 is sufficient small with respect to $\sum_{i=1}^{n} \tau_i$.The processes with transport delay can we obtain by limit $n \to \infty$. All of $\kappa - \tau$ testing systems are included in this set as so as the most of typical industrial processes.

Characteristic numbers of the process

There are many possibilities for system description. The most of methods use three numbers L, τ, K_p obtained from the step response. The Z-N frequency method uses one point of the frequency response, the $\kappa - \tau$ method adds static gain to that.

Our description is based on first three moments of the transfer function

$$m_0 = \int_0^\infty h(t)dt$$

$$m_1 = \int_0^\infty th(t)dt$$

$$m_2 = \int_0^\infty t^2 h(t)dt, \tag{2}$$

where $h(t)$ is impulse response. The numbers $m_0,\ m_1,\ m_2$ can we substitute by $K_0,\ \mu,\ \sigma$ defined as

$$K_0 = \int_0^\infty h(t)dt = m_0$$

$$\mu = \frac{\int_0^\infty th(t)dt}{\int_0^\infty h(t)dt} = \frac{m_1}{m_0}$$

$$\sigma^2 = \frac{\int_0^\infty (t-\mu)^2 h(t)dt}{\int_0^\infty h(t)dt} = \frac{m_2}{m_0} - \frac{m_1^2}{m_0^2}, \tag{3}$$

or equivalently by first three elements of Taylor series of transfer function $F(s)$ at point $s = 0$.

$$\frac{F^{(i)}(0)}{i!} = f_i,\ i = 0, 1, 2,$$

$$f_0 = K_0,\ f_1 = -K_0\mu,\ f_2 = K_0 \frac{\sigma^2 - \mu^2}{2} \tag{4}$$

The restriction of τ value is now useful to define as dependent on measured numbers $K_0,\ \mu,\ \sigma$ as follows

$$\tau_0 \in \langle \alpha_1\mu,\ \alpha_2\mu \rangle,\ \alpha_1, \alpha_2 \in \langle -1, 1 \rangle\ \alpha_1 \le \alpha_2 \tag{5}$$

We choose $\alpha_1 = -0.25, \quad \alpha_2 = 0.5$ for our testing purposes.

Definition 1. Process set model] The transfer function $F(s)$ is admissible, if following conditions are satisfied

(i) (*A priori* admissible systems) Transfer function $F(s)$ is in the form (1), where τ satisfies restrictions (5) and order of the denominator is at most n, where n is given integer.
(ii) (Moment conditions) Transfer function F(s) satisfies conditions (4).

The set of all admissible systems will be called process model set and will be denoted as $\mathcal{S}^n(f_0, f_1, f_2)$.

The following lemma answers the question, for which $n,\ f_0,\ f_1,\ f_2$ the model $\mathcal{S}^n(f_0, f_1, f_2)$ is non-empty. The first lemma considers no zeroes systems ($\alpha_1 = \alpha_2 = 0$).

Lemma 1. The set model $\mathcal{S}^n(f_0, f_1, f_2)$ is non-empty, iff $\frac{1}{n} \le \frac{\sigma^2}{\mu^2} \le 1$

This lemma can be generalized for systems with one zero satisfying (5).

Lemma 2. The set model $\mathcal{S}^n(f_0, f_1, f_2)$ is non-empty, iff $\frac{(1-n)\alpha^2 + 2\alpha + 1}{n} \le \frac{\sigma^2}{\mu^2} \le 1 + 2\alpha_2$,
where $\alpha = argmax_{\alpha_1, \alpha_2}(|\alpha - \frac{1}{n-1}|)$.

For each frequency ω, the value set $\{F(j\omega) : F(s) \in \mathcal{S}^n(f_0, f_1, f_2)\}$ is created by all points $F(j\omega)$, where $F(s)$ ranges over the model set. This region is bounded by finite number of smooth curves, which are created by so called **extreme systems** (for systems with real poles independent on frequency).

Processes on the intersects of smooth curves are suitable for testing methods, because they are lying on the boundary of the value set.

Without loss of generality we can normalize processes in gain and in time to the case $f_0 = 1,\ f_1 = -1$. Lemmas (1) and (2) then determine admissible interval for σ. When $\sigma \to max(resp.\ min)$, we obtain processes close to first-order (resp. n-order) systems as you can see on figure 3.

Testing processes

We have tested all methods on several normalized set models. Lets denote the first group of processes $\{K_0 = \mu = 1,\ \alpha_1 = \alpha_2 = 0,\ \sigma \in (0.35, 0.95)\}$ as $\mathcal{S}_1$ and the second group $\{K_0 = \mu = 1,\ \alpha_1 = -0.25,\ \alpha_2 = 0.5,\ \sigma \in (0.0, 1.3)\}$ as $\mathcal{S}_2$. The order of all testing systems was up to $n = 10$ to avoid numerical errors. The intervals for σ are chosen in according to lemmas (1) and (2).

We obtain the set of all extreme intersect processes for each σ. For model sets in $\mathcal{S}_1$ (nonzero systems) this set can be reduced to only two processes, which create the margin of the band of all frequency responses of set model systems. These two most important processes can we obtain by minimizing or maximizing of the moment $m_3 = \int\limits_0^\infty t^3 h(t) dt$, while the first three moment conditions are satisfied. All extreme intersect processes are in the form

$$F(s) = \frac{\tau_0 s + 1}{(\tau_1 s + 1)^{n_1} (\tau_2 s + 1)^{n_2}} \tag{6}$$

If we consider the case $n \to \infty$ and model set $\mathcal{S}(1, 1, \sigma^2)$, we obtain the following parameterization of extreme transfer functions

$$F_1(s) = \frac{e^{-(1-\sigma)s}}{\sigma s + 1} \tag{7}$$

$$F_1(s) = \frac{1}{(\tau_1 s + 1)^{k-1} (\tau_2 s + 1)} \tag{8}$$

$$k = \lfloor \frac{1}{\sigma^2 - 1} \rfloor$$

$$\tau_1 = \frac{1 + \sqrt{1 + \frac{k}{k-1}(\sigma^2 - 1)}}{k}$$

$$\tau_2 = \frac{1 - (k-1)\sqrt{\frac{k\sigma^s - 1}{k-1}}}{k}$$

3. METHODS TESTING

In this section, the most frequently using methods are tested on models $\mathcal{S}_1$ and $\mathcal{S}_2$. On figure 3 are examples of step responses of testing processes.

Identification

The most of methods use step response to identify three numbers K_p, L, τ (see figure 3). The $\kappa - \tau$ method uses τ as the AB distance derived from $0.63 K_p$. The *Ziegler-Nichols* frequency method uses critical gain and frequency K_u, ω_u for identification (see figure 3). The $\kappa - \tau$ *frequency* method adds the third parameter K_p - static gain.

Fig. 3. Step responses of testing model systems

Fig. 4. Identification in time domain

Fig. 5. Identification in frequency domain

Methods for $1DOF$ *controller*

Methods *IMC, IAE, ISE, ITAE* use serial structure of controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s}\right) \frac{s T_d + 1}{s T_d \alpha + 1}$, where usually $0.05 < \alpha < 0.2$. Methods *IAE, ISE* and *ITAE* have two versions for step response and load disturbance error minimization. Methods

Ziegler-Nichols a *Cohen-Coon* use parallel structure of controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$ The formulas for optimal K_c, T_i, T_d settings for each method are in table 3.

$\kappa - \tau$ *method*

Method $\kappa-\tau$ was presented as an improvement of *Ziegler-Nichols method.* This method works with $2DOF$ controller in a form

$$U = K\left(bY_{sp} - Y + \frac{1}{sT_i}E - \frac{sT_d}{1+sT_d/N}Y\right) \quad (9)$$

and uses two key parameters for identification.

$$\tau = \frac{L}{L+T} \qquad \kappa = \frac{1}{K_p K_u} \quad (10)$$

Normalized values of parameters are then approximated by functions $f(\tau) = a_0 e^{a_1\tau + a_2\tau^2}$ resp. $f(\kappa) = a_0 e^{a_1\kappa + a_2\kappa^2}$. The coefficients a_i can be obtained from tables, and are dependent of the maximal sensitivity value M_s.

Testing criteria

We were looking for maximal overshot, undershot, and time of steady state reaching. The results obtained are putted in two tables for set models $\mathcal{S}_1$(table 4) and $\mathcal{S}_2$ (table 4).

4. CONCLUSION

As acceptable we consider closed loop with maximal overshot 1.2 (steady state after gain normalization is always 1) where the steady state is reached till 5 seconds.

- *IMC* This method is very robust, if our requirements on speed (parameter τ_{cl}) are not very steep. The regulation time is much longer comparing to other methods.
- *IAE-SP, ITAE-SP, ISE-SP* Despite these methods are designed for step response, the requirements are satisfied very rarely. The best of these looks *IAE-SP* which is not unstable for non-zero testing systems.
- *IAE-LD, ITAE-LD, ISE-LD, Ziegler-Nichols, Cohen-Coon* These methods are designed for load disturbance, and that's why the closed loop behavior is unacceptable mainly for one-zero systems. We need to add feed-forward part of the controller to improve it. The best of these looks *ITAE-LD* (stable for set model $\mathcal{S}_1$).
- $\kappa - \tau$ *time* First-order system is not included in $\kappa-\tau$ testing processes, so we can see, that the method fails for processes with $\sigma \rightarrow max$, where is also inclinable to numerical errors.
- $\kappa - \tau$ *frequency* The frequency version gives higher overshot and shorter regulation time. But for $\sigma > 0.7$ can not be simulate because of a second-order system, which appears in extreme processes. The controller is defined for systems with order greater than two.

We can see that each method fails for any type of systems. Each method except *IMC* has problems with processes where $L \rightarrow 0$ (first order systems). For other types of systems, the $\kappa - \tau$ *time* is the best choice from testing methods.

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$f(\tau)$	a_0	a_1	a_2	$f(\kappa)$	a_0	a_1	a_2
aK	3.8	-8.4	7.3	K/K_u	0.33	-0.31	-1.1
T_i/L	5.2	-2.5	-1.4	T_i/T_u	0.76	-1.6	-0.36
T_d/L	0.89	-0.37	-4.1	T_d/T_u	0.17	-0.46	-2.1
b	0.40	0.18	2.8	b	0.58	-1.3	3.5

Fig. 2. $\kappa - \tau$ method coefs for PID tuning, where $M_s = 1.4$

method	Kc	T_i	T_d
IMC	$\frac{1}{K_p}\frac{\tau}{\left(\tau_c l+\frac{L}{2}\right)}$	τ	$\frac{L}{2}$
IAE-setpoint	$\frac{0.65}{K_p}\left(\frac{L}{\tau}\right)^{-1.04432}$	$\frac{\tau}{0.9895+0.09539\left(\frac{L}{\tau}\right)}$	$0.50814\tau\left(\frac{L}{\tau}\right)^{1.08433}$
ITAE-setpoint	$\frac{1.12762}{K_p}\left(\frac{L}{\tau}\right)^{-0.80368}$	$\frac{\tau}{0.99783+0.02860\left(\frac{L}{\tau}\right)}$	$0.42844\tau\left(\frac{L}{\tau}\right)^{1.0081}$
ISE-setpoint	$\frac{0.71959}{K_p}\left(\frac{L}{\tau}\right)^{-1.03092}$	$\frac{\tau}{1.12666-0.18145\left(\frac{L}{\tau}\right)}$	$0.54568\tau\left(\frac{L}{\tau}\right)^{0.86411}$
IAE-load	$\frac{0.98089}{K_p}\left(\frac{L}{\tau}\right)^{-0.76167}$	$\frac{\tau}{0.91032}\left(\frac{L}{\tau}\right)^{1.0521}$	$0.59974\tau\left(\frac{L}{\tau}\right)^{0.89819}$
ITAE-load	$\frac{0.77902}{K_p}\left(\frac{L}{\tau}\right)^{-1.06401}$	$\frac{\tau}{1.14311}\left(\frac{L}{\tau}\right)^{0.70949}$	$0.57137\tau\left(\frac{L}{\tau}\right)^{1.03826}$
ISE-load	$\frac{0.11907}{K_p}\left(\frac{L}{\tau}\right)^{-0.89711}$	$\frac{\tau}{0.7987}\left(\frac{L}{\tau}\right)^{0.9548}$	$0.54766\tau\left(\frac{L}{\tau}\right)^{0.87798}$
Ziegler-Nichols	$\frac{1.2}{K_p\frac{L}{\tau}}$	$2L$	$\frac{L}{2}$
Cohen-Coon	$\frac{\tau}{K_pL}\left(\frac{16\tau+3L}{12\tau}\right)$	$\frac{L\left(32+6\frac{L}{\tau}\right)}{13+8\frac{L}{\tau}}$	$\frac{4L}{11+2\frac{L}{\tau}}$

Table 1. Controller parameters formulas

[redacted]

Table 2. Results for set model $\mathcal{S}_1$

[redacted]

Table 3. Results for set model $\mathcal{S}_2$

Comments

1.05 $overshot$
0.95 $undershot$
5.20 $time[sec]$

1.05	acceptable behavior
1.3	overshot greater than 1.2
→	steady state not reached in 50 seconds
↗	unstable closed loop
–	method can't be applied on these processes