ONE POINT RELAY IDENTIFICATION EXPERIMENT BASED ON CONSTANT-PHASE FILTER

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Abstract: Using standard relay identification methods, one obtains a critical frequency response point with the phase shift -180 degrees. Such phase is not optimal for consequential PI(D) controller design especially when there is no additional knowledge about the process (e.g. static gain). Currently, an adaptive filter is used to get a frequency response sample with different phase shift. Adaptive filter parameters are tuned during the identification experiment. Unfortunately, the adaptation makes the experiment more time consuming. This paper presents a new low-order filter providing arbitrary constant phase at about three decades. Such filter embedded into relay tuner shortens the experiment significantly. Further, the paper discusses several implementation aspect. The authors believe that presented approach may lead to improvement of existing relay autotuners.

Keywords: Relay identification experiment, constant-phase filter, fractional-order integrator

1 INTRODUCTION

Starting from Ziegler-Nichols method [Ziegler and Nichols, 1942], the relay identification experiment became one of the most popular in process control. Although the consequential tuning method is not very reliable [Schlegel, 2002] the tradition to identify a frequency response sample with phase shift -180 degrees survives over the decades [Luyben, 1987; C.C.Hang et al., 2002; H.Huang et al., 1996]. It is caused namely by the natural ability of relay in the feedback to excite the control loop at ultimate frequency corresponding to the phase shift -180 degrees. However, such phase is not suitable for PI(D) controller design especially when there is no additional knowledge about the process – e.g. static gain. In this case, the sample with phase shift -135 degrees is optimal for consequential robust PID controller design [Schlegel, 2000]. For PI controller the proper phase is about -110 degrees. Several methods add to the measured critical point knowledge of the process static gain [Åström and Hägglund, 2006]. Then the phase shift can be up to -220 degrees if the signal/noise ratio and the lag time of the process are big enough.

It flows out from previous facts that there is no universal rule for the phase choice. The optimal phase depends on all process, controller and disturbances in the control loop. Therefore it is highly useful to supply the relay autotuner by a user parameter specifying the required phase. Usually, an adaptive filter is used to get a frequency response sample with different phase shift. Unfortunately, the adaptation makes the experiment very time consuming. This paper presents a new low-order filter providing arbitrary constant phase at about three frequency decades. The filter is based on approximation of reference fractional-order integrator [Hartley and Lorenzo, 2004]. Such filter embedded into relay tuner shortens the experiment significantly.

The paper is organized as follows: In Section 2, some basic properties of relay feedback are reminded. Section 3 describes the new constant phase filter based on reference fractional-order integrator. Section 4 discusses several implementation aspects and proper choice of filter parameters. Illustrative example is given in Section 5. Section 6 contains concluding remarks and ideas for future work.

2 BASIC PROPERTIES OF RELAY FEEDBACK

Relay feedback has attracted significant research attention for more than century. The classical work of Tsypkin [Tsipkyn, 1984] on analysis summarizes the progress till 1960s. In 1950s, relays were mainly used as amplifiers but such applications are obsolete now, because there is the big development of electronic technology. In 1960s, relay feedback was applied to adaptive Control. In 1980s Åström and Hägglund introduced their well known work in which they successfully applied the relay feedback method to auto-tune PID controllers for process control. Since then many researchers have became with new tools and results.

The main advantages of the relay feedback are:

1. It is a closed-loop test. Therefore the process will not drift away from the operating point.
2. For the processes with a long time constant it is a more time-efficient method than conventional step or pulse testing.
3. It identifies the process around the important frequency, for ideal relay at the ultimate frequency with the phase shift -180^o. When we use an appropriate filtr, we can change the phase shift to another more proper value, e.g. for PID controller -135^o.

The relay feedback system is the feedback loop with an ideal (on-off) relay, see Fig. 1a without the adaptive filtr. Consider a relay of magnitude h is inserted in the feedback loop. Initially, the process input $u(t)$ is increased by h. As the output $y(t)$ starts to increase, the relay switches to the opposite value $u(t) = -h$. The close-loop system may start to oscillate with the period P_U and the phase lag is -180^o. The period of the limit cycle is the ultimate period. Therefore, the ultimate frequency from the relay experiment is

$$\omega_U = \frac{2\pi}{P_U}. \tag{1}$$

Using the Fourier series expansion, the periodic $u(t)$ can be written as

$$u(t) = \frac{4h}{\pi}\sum_{n=0}^{\infty}\frac{\sin((2n+1)\omega t)}{2n+1}. \tag{2}$$

The amplitude a of the process output $y(t)$ can be considered to be the result of the primary harmonic of the relay output. Therefor, the ultimate gain can be approximated as [Åström and Hägglund, 1984]

$$K_U = \frac{4h}{\pi a}, \tag{3}$$

where h is the height of the relay and a is the amplitude of oscillation. With a ideal relay, a small amount of noise can make the relay switch randomly. By introducing hysteresis, the noise must be larger than the hysteresis width to make the relay switch. For the relay with hysteresis, the ultimate gain can be approximated as

$$K_U = \frac{4h}{\pi\left(\sqrt{a^2-\epsilon^2} - j\epsilon\right)}, \tag{4}$$

where h is the height of the relay, a is the amplitude of oscillation and ϵ is the relay hysteresis width.

Further, we use this technique to compute the amplitude of the process frequency response point. The phase is shifted from the origin -180^o using a constant-phase filter described in the next section.

3 CONSTANT-PHASE FILTER

The identification of sample with arbitrary phase shift is always based on relation

$$\arg P(\mathrm{j}\omega_U) + \arg F(\mathrm{j}\omega_U) = -180 \ \ [\mathrm{deg}], \tag{5}$$

where $P(s)$ is the process, $F(s)$ is the suitable filter added to the relay feedback and ω_U is the frequency of steady closed loop oscillations.

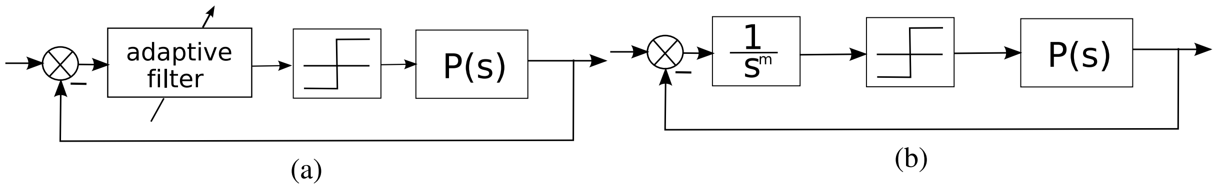


Figure 1 – Relay with (a) adaptive filter, (b) constant-phase filter.

3.1 Traditional solution

One way to obtain the frequency sample with arbitrary phase shift is to complete the relay by a special adaptive low-order filter $F(s)$ (Fig. 1a). The filter parameters are changed during the relay experiment in order to reach the proper phase shift. Such adaptation requires some advanced numerical optimization method and increases the computational burden. Moreover, the adaptation makes the experiment very time consuming namely for slow temperature or chemical processes.

3.2 New approach

It flows out from previous subsection, that the filter $F(s)$ with constant phase shift in the entire frequency band will speed up identification significantly. For simplicity, the phase shift of the filter must be determined by one user parameter. Fractional-order integro-differential operator described by irrational transfer function

$$F(s) = \frac{1}{s^m}, \quad m \in \mathbf{R} \tag{6}$$

fulfills our requirements. Frequency response of (6) can be computed by substituting $s = \mathrm{j}\omega$ as

$$F(\mathrm{j}\omega) = \frac{1}{(\mathrm{j}\omega)^m} = \frac{1}{\omega^m}(-\mathrm{j})^m = \frac{1}{\omega^m}\left(\cos\frac{\pi}{2}m - \mathrm{j}\sin\frac{\pi}{2}m\right). \tag{7}$$

Hence, the frequency response in complex plane is a straight line going through a coordinate origin (Fig. 2) and the constant-phase property is ensured. Assuming (5) and (7) the parameter m is related to required phase shift φ as

$$m = 2 - \varphi/90. \tag{8}$$

In the next section, the continuous approximation of irrational transfer function (6) will be discussed.

3.3 Optimization of filter parameters

It is impossible to realize the transfer function (6) precisely in the entire frequency band by finite dimensional filter. One has to restrict on a given frequency interval (ω_L, ω_H). Assume, that the approximation of (6) has a zero/pole form

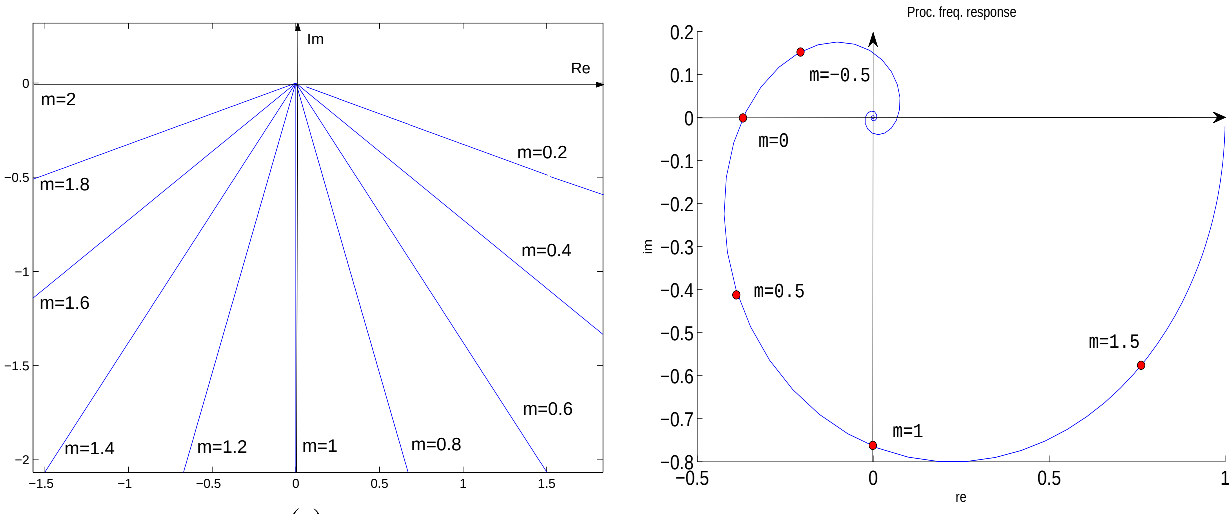
$$\frac{1}{s^m} \approx \widehat{F}(s) = 10^{K_0} \frac{\prod_{i=1}^{M}(10^{-\omega_{Z_i}}s+1)}{\prod_{i=1}^{N}(10^{-\omega_{P_i}}s+1)}. \tag{9}$$

Then the optimality criterion can be defined as

$$J = \int_{\omega_L}^{\omega_H} |\widehat{F}(\mathrm{j}\omega) - (\mathrm{j}\omega)^{-m}|^2 \mathrm{d}\omega. \tag{10}$$

Standard MATLAB numerical procedure `fmincon` can be used for optimization of parameter vector

$$\mathbf{x} = [K_0, \omega_{P_1}, \ldots, \omega_{P_N}, \omega_{Z_1}, \ldots, \omega_{Z_M}]. \tag{11}$$



(a) (b)

Figure 2 – (a) Fractional integrator frequency responses, (b) sample phase shift for different m.

In Figure 3(a) are depicted the optimized frequencies corresponding to zero/pole positions for $M = N = 4$. Except the upper and lower bound of frequency interval, the zeros/poles positions depending on m can be almost interpolated by linear function of m as arose originally from [Charef et al., 1992]. However, for low order approximations ($M, N \leq 5$) is the declination from linear shape significant. Therefore, the following relation must be used for interpolation.

$$
\begin{aligned}
K_0 &= 2.0301\ \mathrm{e}^{-0.0274m+1.5077m^2-2.8068m^3+1.8906m^4} \\
Z &= 2.9158\ \mathrm{e}^{0.0272m+1.2249m^2+2.2259m^3+1.4754m^4} \\
P &= 2.8822\ \mathrm{e}^{-0.1987m+0.0310m^2-0.0254m^3+0.0060m^4} \\
\omega_{Z1} &= Z \\
\omega_{Z2} &= -P+3 \\
\omega_{Z3} &= 1.0599+0.4435m-0.0050m^2 \\
\omega_{Z4} &= 1.9401+0.4436m+0.0049m^2 \\
\omega_{P1} &= P \\
\omega_{P2} &= -Z+3 \\
\omega_{P3} &= 1.0599-0.4435m-0.0050m^2 \\
\omega_{P4} &= 1.9401-0.4436m+0.0049m^2
\end{aligned}
\tag{12}
$$

The relations (12) define the filter (9) for $\omega_L = 10^0$ and any $m \in (0, 1)$. Thanks to the symmetry, the filter for $m \in (-1, 0)$ can be gained by switching zeros to poles and vice versa.

Note, that the maximum phase error is obtained for $m = \pm 0.5$. It can be checked in Fig. 3 that the phase difference between $\widehat{F}(s)$ and the required one is at most 1 degree.

4 IMPLEMENTATION ASPECTS

Let us summarize the key implementation aspects which are necessary to have in mind.

4.1 Proper choice of ω_L

Although the filter gives constant phase in the 3 decades band, it is important to make ω_L close to the process ultimate frequency ω_{CP}. Otherwise the filter $F(s)$ dynamics will be slow with respect to

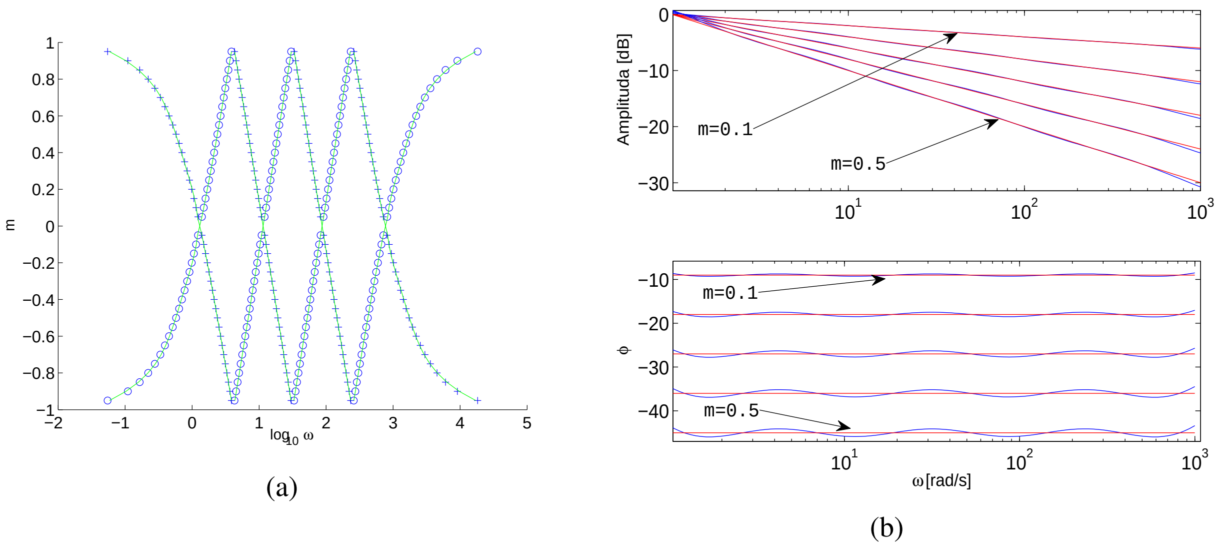


Figure 3 – (a) $\widehat{F}(s)$ zeros/poles positions for $M = N = 4$, $\omega_L = 10^0$, $\omega_H = 10^3$ [rad/s], (b) Bode characteristics of $\widehat{F}(s)$ (red) and $F(s)$ (blue).

the process and the time of reaching steady oscillations will increase. In practice, the user will roughly specify the dynamics of the process (e.g. time to reach steady state). Thus, one needs a way to recompute the filter parameters (12) for arbitrary ω_L or arbitrary central frequency ω_C. The transformation relations for 3-decades filter are

$$\widehat{\omega}_* = \omega_* + \log_{10}\omega_L = \omega_* + \log_{10}\omega_C - 1.5, \quad * = \{P_1, \ldots, P_N, Z_1, \ldots, Z_M\}, \tag{13}$$

$$\widehat{K_0} = K_0 - m\log_{10}\omega_L = K_0 - m(\log_{10}\omega_C - 1.5). \tag{14}$$

4.2 Proper choice of m

Let's assume that the process gain is known, e.g. we are limited in the phase choice (determined by m) only by the process dynamics and noises in the measured signal. Naturally, for processes close to first order, the point with higher phase shift cannot be gained on principle. On the contrary, for dead-time dominant processes the phase shift can be increased significantly.

4.3 Proper choice of sampling time

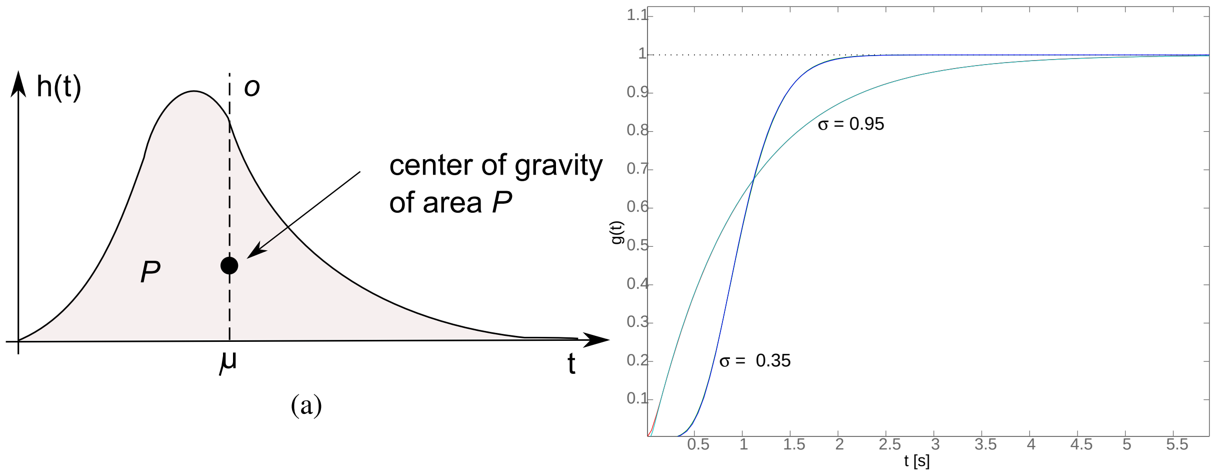
The maximum possible sampling time also depends of the process time scale and also on the normalized dead time.

4.4 Noises in the measured signal

Big noises in the measured signal may damage the experiment at higher frequencies with the phase shift above 180 degrees. Therefore, the phase must be chosen also according to the noises in measured signal.

4.5 Rules for filter parameters choice

It is evident from the previous subsections, that one has to characterize the process gain, time-scale, and normalized dead time to develop rules for the choice of filter parameters. Moment model set approach can serve for such purpose [Vecerek, 2004]. Let us assume that the process is described by first



(a) (b)

Figure 4 – (a) Interpretation of three moments of the impulse response, (b) Process dynamics influenced by parameter σ^2.

three moments m_1, m_2, m_3 of the impulse response $h(t)$ defined as

$$m_i = \int_{t=0}^{\infty} t^i h(t)\mathrm{d}t, \quad i = 0, 1, 2 \tag{15}$$

or equivalently by more suitable group of numbers κ, μ, σ^2 defined as follows

$$\kappa = m_0, \qquad \mu = \frac{m_1}{m_0}, \qquad \sigma^2 = \frac{m_2}{m_0} - \frac{m_1^2}{m_0^2}. \tag{16}$$

The numbers κ, μ, σ^2 have the required interpretation as they define the process gain, time scale and normalized dead time, respectively (see Fig. 4a). Further it is possible to restrict ourselves to normalized values for gain and time, thus $\kappa = 1, \mu = 1$. Then, process dynamics varies from first-order to pure dead time depending on one parameter σ^2 as depicted in Fig. 4b.

The high percentage of processes has a normalized dead-time σ^2 in the interval $(0.6, 0.95)$. Let us denote this class of typical processes as Σ_1. As the 3 decades filter cannot cover complete interval of normalized dead time it is necessary to define another two groups of processes. Let us denote the processes close to first order and pure dead time as Σ_2 and Σ_3, respectively. Table 1 shows the proper choice of filter central frequency ω_C with respect to the required phase for the normalized value $\mu = 1$. The intervals of σ^2 for which the phase error of identified point is less than 2 degrees are summarized in this table as well. The identification was tested for large variations of $\mu \in (0.1, 10)$ which correspond to variations of process settling time $t \in (T_{Cmin}, T_{Cmax})$.

It leads to the assumption that the user must be only able to specify the approximate process settling time T_C for example as outlined in table 2. Note, that for $\mu = 1$ the approximate settling time is about 10 s. Finally, the corresponding filter central frequency can be simply obtained as $\widehat{\omega_C} = \omega_C/\mu$, where ω_C is the frequency taken from table 1.

5 ILLUSTRATIVE EXAMPLE

Let us choose the following example to illustrate the proposed method. Consider the transfer function of the second order plus dead time system

$$P(s) = \frac{1}{(\tau_1 s + 1)(\tau_2 s + 1)}\ e^{-Ds} = \frac{1}{(15s + 1)(s + 1)}\ e^{-0.5s}. \tag{17}$$

Table 1 – Rules for choice of filter central frequency ω_C for normalized $\mu = 1$.

$\Sigma \backslash \varphi$		110^o	135^o	160^o	200^o	220^o
Σ_1	σ^2	–	$0.6 - 0.95$	$0.65 - 0.92$	$0.65 - 0.94$	–
	ω_C		13.85	22.28	31.46	
Σ_2	σ^2	$0.85 - 0.98$	$0.85 - 0.98$	$0.9 - 0.98$	–	–
	ω_C	16	33.76	82.34		
Σ_3	σ^2	–	–	$0.3 - 0.8$	$0.3 - 0.8$	$0.3 - 0.8$
	ω_C			10.16	11.73	15.26

Table 2 – Choice of μ with respect to the process settling time T_C.

μ	T_{Cmin}	T_C	T_{Cmax}
0.1	0.1 s	1 s	10 s
6	6 s	1 min	10 min
360	6 min	1 h	10 h
8640	2.5 h	1 day	10 days

The transfer function (17) has these three characteristic numbers [Vecerek, 2004]

$$\kappa = 1, \quad \mu = \tau_1 + \tau_2 + D = 16.5, \quad \sigma^2 = \tau_1^2 + \tau_2^2 = 226. \tag{18}$$

After normalization in time we get

$$\kappa_N = 1, \quad \mu_N = 1, \quad \sigma_N^2 = \frac{\sigma^2}{\mu^2} = 0.83. \tag{19}$$

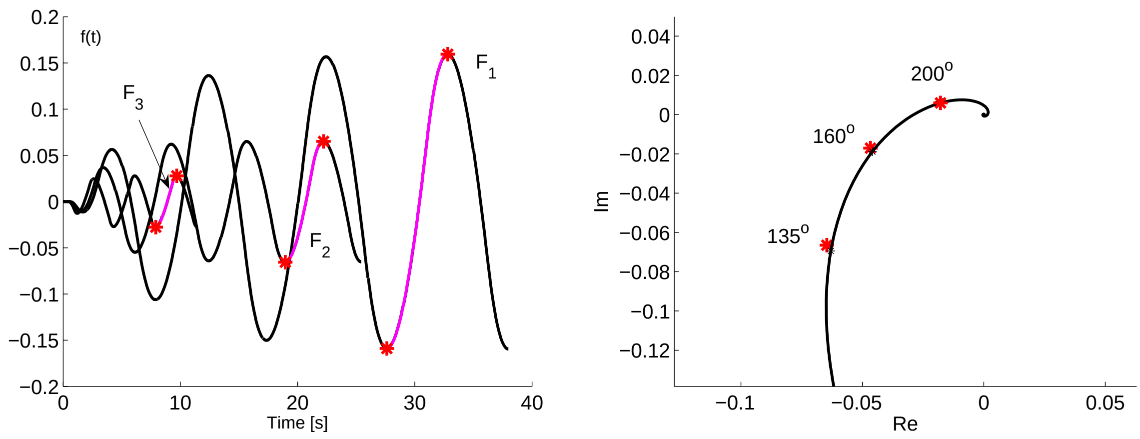
The system (17) belongs to the group Σ_1 from the table 1. Further, we configure the constant-phase filter (CPF) for three phase lags. The proper values of the central frequency ω_C are $\omega_{C1} = 13.85$, $\omega_{C2} = 22.28$ and $\omega_{C3} = 31.46$ for the phase lags $\varphi_1 = 135^o, \varphi_2 = 160^o$ and $\varphi_3 = 200^o$ from the table 1, respectively. The phase lags φ_1, φ_2 and φ_3 correspond to the CPF powers (6) $m_1 = 0.5, m_2 = 0.2222$ and $m_3 = -0.2222$.

For CPF transfer functions it holts from (12) and (9)

$$\begin{aligned} F_1(s) &= \frac{0.000007285s^4 + 0.006125s^3 + 0.4134s^2 + 3.402s + 3.108}{0.0003158s^4 + 0.06627s^3 + 1.544s^2 + 4.385s + 1}, \\ F_2(s) &= \frac{0.000002035s^4 + 0.001665s^3 + 0.1331s^2 + 1.321s + 1.417}{0.00001148s^4 + 0.005311s^3 + 0.2657s^2 + 1.649s + 1}, \\ F_3(s) &= \frac{0.000002201s^4 + 0.001437s^3 + 0.1015s^2 + 0.89s + 0.762}{0.0000003612s^4 + 0.0004175s^3 + 0.04713s^2 + 0.6601s + 1}. \end{aligned} \tag{20}$$

The simulation is made in discrete time with the sampling period $T_S = 0.01s$. Therefor, it is necessary to make discretization of the CPF transfer functions (20)

$$\begin{aligned} F_{z1}(z) &= \frac{0.07091z^4 - 0.1304z^3 + 0.03358z^2 + 0.04555z - 0.01956}{z^4 - 2.805z^3 + 2.649z^2 - 0.8767z + 0.03284}, \\ F_{z2}(z) &= \frac{0.3087z^4 - 0.5449z^3 + 0.1238z^2 + 0.1824z - 0.06969}{z^4 - 2.197z^3 + 1.239z^2 + 0.1509z - 0.1924}, \\ F_{z3}(z) &= \frac{3.245z^4 - 6.187z^3 + 2.192z^2 + 1.442z - 0.6897}{z^4 - 1.47z^3 - 0.0499z^2 + 0.692z - 0.1698}. \end{aligned} \tag{21}$$



(a) (b)

Figure 5 – (a) The filter output oscilations, (b) process nyquist plot with computed points.

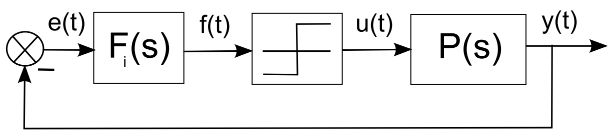


Figure 6 – The relay feedback simulation scheme.

The simulation scheme is shown in Figure 6. The resulting oscillations from the CPF output $f(t)$ are depicted in Figure 5 (a). The simulation is stopped, when the variance in amplitude between two peaks is less then 1%. The peaks used to compute the amplitude a are marked by the red star in Figure 5 (a). The relay amplitude is set to $h = 1$. The ultimate gain of the system $P(s)$ with the CPF can be computed using equation (4). For the frequency points $G(\mathrm{j}\omega_U)$ it holds

$$G_i(\mathrm{j}\omega_{Ui}) = F_i(\mathrm{j}\omega_{Ui})P(\mathrm{j}\omega_{Ui}) = -\frac{1}{K_{Ui}} = -\left(\frac{\pi}{4h}\sqrt{a_i^2 - \epsilon^2} - \mathrm{j}\frac{\pi\epsilon}{4h}\right),\ i = 1, 2, 3. \quad (22)$$

Further, the resulting frequency response points depicted in Fig. 5 (b) are computed as

$$P(\mathrm{j}\omega_{Ui}) = \frac{G_i(\mathrm{j}\omega_{Ui})}{F_i(\mathrm{j}\omega_{Ui})},\ i = 1, 2, 3. \quad (23)$$

The results are summarized in the table 3.

6 CONCLUSION

In this paper an advanced relay identification experiment was presented. Using new constant-phase filter one can identify the frequency response point with arbitrary phase shift without any adap-

Table 3 – The results of the illustrative example.

	Amplitude	Phase shift [deg]	ω_U [rad/s]
φ_1	0.09277863	134.1801	0.6047
φ_2	0.04967579	159.9205	0.9652
φ_3	0.01937714	200.333	1.7551

tation. The filter parameters were numerically optimized with respect to fractional integrator reference model. Further, several implementation aspects were discussed. The paper concentrates especially to proper choice of the frequency band of filter realization. Finally, the illustrative example was shown. The authors believe that the proposed technique can find a lot of applications in compact relay auto-tuners.

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