Stability regions for PI/PID controller and Matlab program

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Abstract:
The systematic procedure for construction of robustness regions in the parameter plane is given in the general case of stable and unstable as well as irrational systems with time delay. Design specifications are assumed in the form of simple constraints on the sensitivity and complementary sensitivity functions. The developed method was implemented in Matlab GUI for common used controller types - PID, PI, L-L. It is believed that the developed tool may be very useful for design and tuning of industrial controllers.

Key words: robust control, parametric plane method, PID control, computer aided design

1 Introduction

The adjustable parameters of a regulator for a linear system are tuned by different methods, for example by the root locus technique, optimal design and time or frequency analysis. Frequency methods are widely used because they have a simple physical interpretation and are applicable to a wider class of systems such as non-minimum phase systems, unstable systems or system with significant time delay.

The design technique presented in this paper is a graphical approach based on the method of D-partition. This technique allows direct design of PI(D) controllers for which the open-loop Nyquist plot fulfill the restrictions on the sensitivity and complementary sensitivity functions. The restrictions are maximal values of sensitivity and complementary sensitivity functions in the defined frequency intervals. These specifications can be converted to circle constraints for the open-loop Nyquist plot.

The proposed method isolates the sensitivity and complementary sensitivity regions in the parameter plane. The aim of this paper is to extend the design technique [1], and provide the specified disturbance rejection at a given frequency range and the proper bandwidth of the closed loop.

The paper is organized as follows. In Section 2, the systematic method for construction of the robustness regions in parameter plane is presented. Several examples of PID controller design solved in Matlab GUI are given in Section 3. Conclusions will be drawn in Section 4.

2 Problem statement and PID control design

Let $P(s)$ represents the process and $C(s)$ the PID controller with transfer function

$$C(s) = k\left(1 + \frac{1}{T_i s} + \frac{T_d s}{\frac{T_d}{N} s + 1}\right) = k + \frac{k_i}{s} + \frac{k_d s}{\frac{k_d}{N k} s + 1} \quad , \tag{1}$$

where k, T_i and T_d are its engineering parameters and $k_i = k / T_i$, $k_d = kT_d$ are gains used in the following. The open loop transfer function $L(jw)$, the sensitivity function $S(jw)$ and the complementary sensitivity function $T(jw)$ are, respectively, defined by

$$L(jw) = C(jw)P(jw), \quad S(jw) = \frac{1}{1 + L(jw)} \; , \quad T(jw) = \frac{L(jw)}{1 + L(jw)} \; .$$

The maximum sensitivity M_S and the maximum complementary sensitivity M_T are, respectively, defined by

$$M_S = \sup_w |S(jw)| \; , \quad M_T = \sup_w |T(jw)| \; . \tag{2}$$

The explicit parametric curves associated with (2) are given in [1]. These curves describe the boundaries of robustness regions in the parameter plane k-k_i of the PI(D) controller. Points on the boundary of the regions are PID gains for which the Nyquist open-loop plot is tangential to a circle corresponding to M_S or M_T .

In this paper, we consider specifications depicted in Fig. 1. Except (2), we require the maximum of the sensitivity function up to the frequency w_S is lower then $e_S > 0$ and the maximum of the complementary sensitivity function over the frequency w_T is limited by $e_T > 0$,

$$\sup_{w \in <0, w_s>} |S(jw)| \le e_S \; , \tag{3}$$

$$\sup_{w \in <w_T, +\infty)} |T(jw)| \le e_T \; . \tag{4}$$

Similarly as in [1], we restrict the PID gain finding problem to the case of two design parameters k and d for which it holds

$$k_i = kd \ , \qquad k_d = k\frac{f}{d} = \frac{fk^2}{k_i} \ , \tag{5}$$

where $d = 1/T_i$ and f is a fixed parameter with the meaning $f = T_d / T_i$. Classically it was quite common to postulate $f = 1/4$.

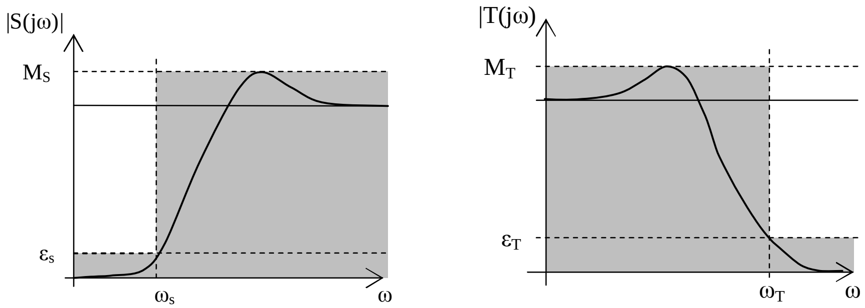


Fig. 1 The constrains on sensitivity and complementary sensitivity functions

In order to determinate the (k, d) values for which the specifications (2), (3) and (4) are satisfied, consider the following notation

$$L(jw) = \ u + jv,\ C(jw) = x + jy,\ P(jw) = a + jb, \tag{6}$$

Design specification (3) is equivalent to the condition that the open-loop Nyquist plot $L(jw),\ w \in < 0, w_S >$ is outside the disk $U(c_s, r_s)$ with the center $c_S = -1$ and radius $r_S = 1/e_S$ and similarly design specification (4) to that the Nyquist plot $L(jw),\ w \in < w_T, +\infty)$ is outside the disk $U(c_T, r_T)$ with the center $c_T = e_T{}^2/(1\text{-}e_T{}^2)$ and radius $r_T = e_T / \left| e_T{}^2 - 1 \right|$.

Now, it is clear that the pairs (k, d) on the boundaries of the sensitivity function regions corresponding to (3) satisfy the following conditions

$\left| S(jw_S) \right| = e_S$,

$L(jw)$ is tangential to the boundary of $U(c_s, r_s)$ at some $w \in < 0, w_S >$.

Or, equivalently

$$(u(w_S) - c_S)^2 + v(w_S)^2 = r_S^2 ,$$
$$(u(w) - c_S)u'(w) + v(w)v'(w) = 0 \quad \text{for some} \quad w \in <0, w_S>.$$

Similarly, for complementary sensitivity function regions corresponding to (4) we obtain

$$(u(w_T) - c_T)^2 + v(w_T)^2 = r_T^2 \quad ,$$
$$(u(w) - c_T)u'(w) + v(w)v'(w) = 0 \quad \text{for some} \quad w \in < w_T, +\infty).$$

3 Examples solved in Matlab GUI

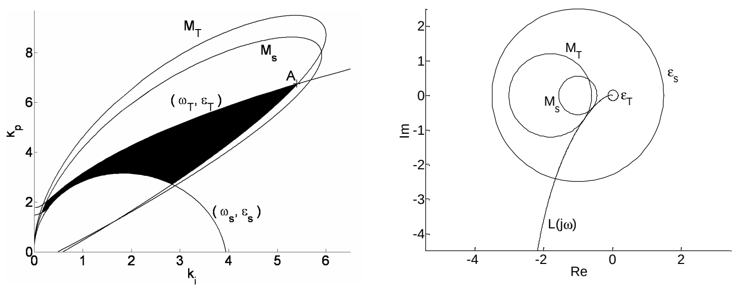
Example 1. Consider the following process

$$F(s) = \frac{1}{(0.2s+1)(0.7s+1)(0.9s+1)} .$$

It is required to find the regions of PID gains (k_p, k_i) for the specifications

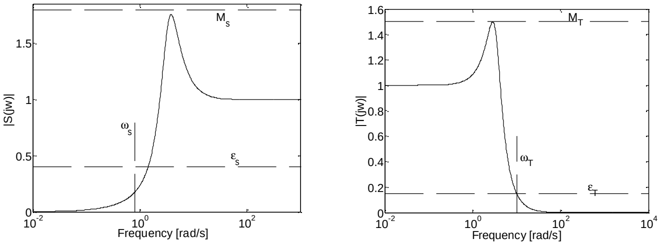
$$M_s = 1.8, \quad w_s = 0.8, \quad e_s = 0.4, \qquad M_T = 1.6, \quad w_T = 10, \quad e_T = 0.5 \quad .$$

The corresponding regions are depicted in Fig. 2a. The resulting region is shaded. The Nyquist plot for the point A is shown in Fig. 2b. In Fig. 3 are shown the corresponding sensitivity and complementary sensitivity functions.



(a) (b)

Fig. 2 (a) the region of robustness stability; (b) open-loop Nyquist plot



(a) (b)

Fig. 3 (a) the sensitivity function; (b) the complementary sensitivity function

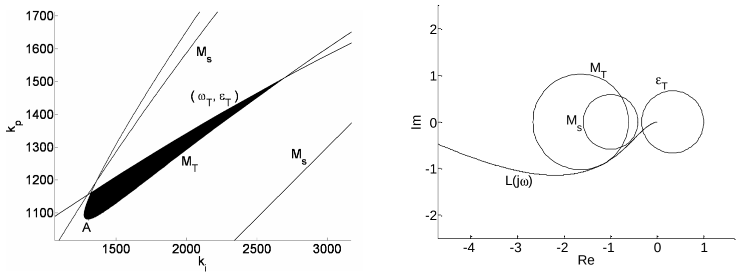
Example 2. Consider the following process

$$F(s)=\frac{1}{s}\left[\frac{17.8}{142.86s+1}+\frac{0.01*8^2}{s^2+2*0.015*8s+8^2}\right].$$

It is required to find the regions of PID gains (k_p, k_i) satisfying the following design specifications

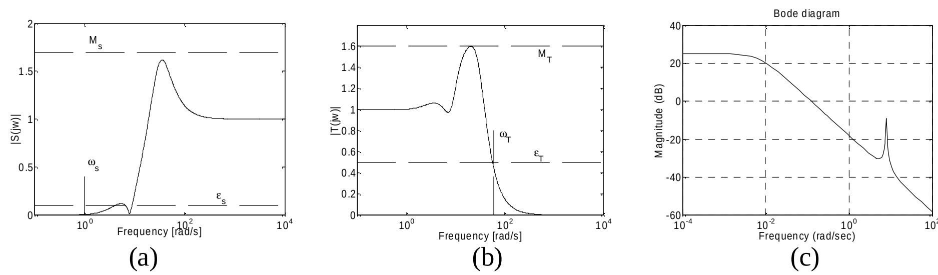
$$M_s = 1.8, \quad w_s = 1.0, \quad e_s = 0.1, \quad M_T = 1.6, \quad w_T = 55, \quad e_T = 0.5 \quad .$$

The results of design are presented in Fig. 2-3.



(a) (b)

Fig. 2 (a) the robustness region; (b) the open-loop Nyquist plot for point A



(a) (b) (c)

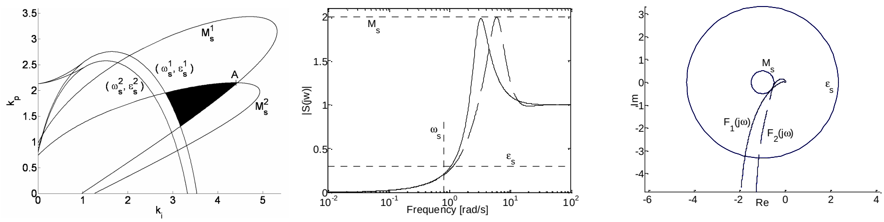
Fig. 3 (a) the sensitivity function; (b) the complementary sensitivity function; (c) Bode diagram for $sF(s)$

Example 3. To illustrate the design of a robust PID controller for a finite number of systems, consider the process models

$$F^1(s) = \frac{1}{(0.2s+1)(0.4s+1)^2}, \quad F^2(s) = \frac{1}{(0.5s+1)(0.08s+1)^5}$$

and the design specifications

$$M_s = 1.8, \; w_s = 0.8, \; e_s = 0.4.$$



(a) (b) (c)

Fig. 5 (a) the stability region; (b) the sensitivity functions; (c) the Nyquist plots

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4 Conclusion

The practical robust design problem of the PID controller was formulated and solved by the method of robustness regions. The explicit equations are provided for determining the boundaries of the corresponding robustness regions. The developed method may be used for any plant dimension including pure delay and unstable plants. The outcome of applying the design algorithm is a complete set of controllers consistent with the design specifications. The authors believe that the developed Matlab program may be very useful for practitioners.

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