Exact Revision of the Ziegler–Nichols Frequency Response Method

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ABSTRACT
The main subject of this paper is to provide an exact theoretically-based revision of the Ziegler–Nichols frequency-response method by solving the newly formulated robust design problem. It is shown that the original method can be considerably improved if a suitable point on the Nyquist curve is used instead of the ultimate point.

KEY WORDS
Process Control, Robust Control; Non-Convex Optimization; PID Control; Autotuners

1 Introduction

In the celebrated paper of Ziegler and Nichols [1], the authors promised that mathematical derivations of their empirical formulae would be published at a later time when convenient. In spite of the great popularity of the Ziegler–Nichols (ZN) methods in industry as well as in the academic sphere, it seems that something like that is still lacking. Although there exist a large number of works concerning the ZN methods [2], [3], all these are empirical as the original paper. The main subject of this contribution is to provide a strictly exact theoretically-based revision of the ZN frequency response method by solving a nonstandard robust design problem which has been motivated by recent works on worst-case identification [4], [5], [6], non-convex optimization [7] and the value set concept [8].

Notation and Preliminaries: Let the field of real numbers be denoted by $\mathbb{R}$, the interval of real numbers $(-\infty, 0]$ by $\mathbb{R}^-$ and the complex plane by $\mathbb{C}$. If A is a subset of $\mathbb{C}$, then let $A^\circ, \partial A$ and $\mathrm{cl}A$ denote the interior, the boundary and the closure of the set A, respectively. Let $z \cdot A$ denote the product of the complex number z and the set $A \subset \mathbb{C}$ defined by $z A = \{w = z a : a \in A\}$. Clearly, $\partial(z A) = z \partial A$. Let $\deg p(s)$ denote the degree of the polynomial $p(s)$. Let $\arg f(j\omega)$ denote the total change in the angle of the complex function $f(j\vartheta)$ when ϑ varies from 0 to ω. A set X is said to be *pathwise connected* if the following condition holds: Given any two points $x_0, x_1 \in X$, there exists a continuous function $\Phi : [0, 1] \to X$ such that $\Phi(0) = x_0$ and $\Phi(1) = x_1$.

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2 Problem Formulation

Consider a control loop with the classical PID controller specified by transfer function

$$G(s) = K\left(1 + \frac{1}{T_i s} + \frac{T_d s}{\frac{T_d}{N} s + 1}\right) \qquad (1)$$

where K is the proportional gain, T_i is the integral time and T_d is the derivative time. It is assumed that the derivative part is filtered by the first-order filter with the time constant T_d/N, where N is a fixed parameter. Our aim is to design the free parameters $K > 0$, $T_i > 0$, $T_d > 0$ in such a way that the closed-loop system satisfies some robust performance specification for all unfalsified plant models which are consistent with given *a priori* information and with one sample of the plant frequency response. To formulate this problem in a precise manner, some definitions are needed.

Definition 1 (Unfalsified Plant) *A plant transfer function $F(s)$ is said to be unfalsified if the following two conditions hold:*

(i) (A priori Hypothesis) The transfer function $F(s)$ is in the form

$$F(s) = \frac{1}{p(s)}$$

where $p(s)$, $\deg p(s) \le n$, is a polynomial with real nonnegative coefficients, and all roots of $p(s)$ lie in the interval $\mathbb{R}^- = (-\infty, 0]$.

(ii) (Experimental Data Interpolation) Nyquist plot $F(j\omega), \omega > 0$ is passing (in the first circulation) through the given point F_1 at the frequency ω_1. More formally

$$F(j\omega_1) = F_1, \qquad -2\pi < \arg F(j\omega_1) < 0.$$

Furthermore, the set of all unfalsified transfer functions is called the unfalsified plant family and denoted by $S^n_{\mathbb{R}^-}(F_1, \omega_1)$.

If the clause that the polynomial $p(s)$ has just k, $1 \le k \le n$, roots equal to 0 is added to condition (i), then the corresponding unfalsified family is denoted by $S^n_{\mathbb{R}^-,k}(F_1, \omega_1)$.

The condition (i) expresses the *a priori* hypothesis that the plant can be described with sufficient precision by an all poles stable overdamped transfer function. Note that in the limit case ($n \to \infty$) this description covers any transfer function in the form

$$F(s) = \frac{K_0 e^{-Ds}}{p(s)},$$

where $D > 0$ and $p(s)$ is an arbitrary polynomial with roots in the interval $(-\infty, 0]$. This form seems to be quite general for the process control field.

Definition 2 *(Ziegler–Nichols Experiment Based Robust Design Problem). We are given one disturbance-free sample of the plant frequency response F_1, ω_1 and a fixed n. Find the parameters of the PID controller (1) that minimize $\frac{T_i}{K}$ subject to the constraints that the Nyquist curve $L(j\omega) \triangleq G(j\omega)F(j\omega)$ satisfies the encirclement (stability) condition and that it is outside a circle with center at $s = C$ and radius R (i.e. for all ω and for any unfalsified transfer function $F(s) \in S^n_{\mathbb{R}^-}(F_1, \omega_1)$ it holds $L(j\omega) \notin U(C, R)$, where $U(C, R) \triangleq \{s \in \mathbb{C} : |s - C| < R\}$).*

Note that the criterion

$$I \triangleq \frac{T_i}{K} \to \min$$

is equivalent to the requirement that the integrated error due to a load disturbance in the form of a unit step at the plant input is minimized.

The constraint that the Nyquist curve is outside the circle expresses the condition on the largest value of the sensitivity or complementary sensitivity function. More formally, if the limitation

$$\sup_{\omega} \left| \frac{1}{1 + L(j\omega)} \right| \le M_s$$

of the sensitivity function is required, then

$$C = -1, \qquad R = \frac{1}{M_s}. \tag{2}$$

Similarly, if the limitation

$$\sup_{\omega} \left| \frac{L(j\omega}{1 + L(j\omega)} \right| \le M_p$$

of the complementary sensitivity function is required, then

$$C = \frac{M_p^2}{1 - M_p^2}, \qquad R = \frac{M_p}{|M_p^2 - 1|}. \tag{3}$$

Further extensive discussion on these design specifications can be found in [7].

It is evident that for obtaining a numerical solution of the above optimization problem (Definition 2) some parametrization of the unfalsified plant family is needed. However, in the next section it is shown that the parametrization of only some subset of all (in some sense) ”ultimate” transfer functions is sufficient.

3 Ultimate Plant Set Parametrization

Definition 3 *(Value Set of Unfalsified Plant Family). We call*

$$\mathcal{F}^n_{\mathbb{R}^-}(F_1, \omega_1; \omega) = \{F(j\omega) : F(s) \in S^n_{\mathbb{R}^-}(F_1, \omega_1)\}$$

the value set of the unfalsified plant family $S^n_{\mathbb{R}^-}(F_1, \omega_1)$ at frequency $\omega \ge 0$.

Roughly speaking, given unfalsified plant family $S^n_{\mathbb{R}^-}(F_1, \omega_1)$, then, at a fixed frequency $\omega \ge 0$, the value set is the subset of the complex plane consisting of all the values which can be assumed by $F(j\omega)$ as $F(s)$ ranges over $S^n_{\mathbb{R}^-}(F_1, \omega_1)$. It will be proved that the value set $\mathcal{F}^n_{\mathbb{R}^-}(F_1, \omega_1; \omega)$ for $\omega \ge 0$ is bounded by a connected boundary, which consists of a finite number of smooth arcs.

Definition 4 *(Ultimate Transfer Function of the Unfalsified Plant Family). An unfalsified transfer function $F(s) \in S^n_{\mathbb{R}^-}(F_1, \omega_1)$ is said to be ultimate if there exists at least one frequency $\omega > 0, \omega \ne \omega_1$, such that $F(j\omega) \in \partial\mathcal{F}^n_{\mathbb{R}^-}(F_1, \omega_1; \omega)$.*

To explain the purpose of the ultimate transfer functions, consider a unity feedback system with an open loop $L(s) \triangleq G(s)F(s)$, where $G(s)$ is a fixed stable controller and $F(s)$ ranges over the family $S^n_{\mathbb{R}^-}(F_1, \omega_1)$. Now, let P denote the property (Definition 2) that Nyquist curve $L(j\omega)$ satisfies the encirclement condition and that

$$\forall \omega > 0 : L(j\omega) \notin U(C, R),$$

where $U(C, R)$ is an open disc such that $-1 \in U(C, R)$. Under these assumptions, the following lemma holds.

Lemma 1 *Suppose that $L(0) \notin U(C, R)$ for all $F(s) \in S^n_{\mathbb{R}^-}(F_1, \omega_1)$. Then the property P holds for all $F(s) \in S^n_{\mathbb{R}^-}(F_1, \omega_1)$ if and only if the property P holds for all ultimate transfer functions.*

Proof: Since an ultimate transfer function is unfalsified, the necessity is trivial. To establish sufficiency, we consider two cases:
(i) Firstly, we assume that there exists an unfalsified transfer function $F_1(s)$ and frequency $\hat{\omega} > 0$ such that $L_1(j\hat{\omega}) \triangleq G(j\hat{\omega})F_1(j\hat{\omega}) \in U(C, R)$. Since we assume that $L_1(j0) \notin U(C, R)$, by continuity $L_1(j\omega)$, it follows that there exists $\tilde{\omega}, \tilde{\omega} < \hat{\omega}$, such that some boundary point z of the set $G(j\tilde{\omega})\cdot\mathcal{F}^n_{\mathbb{R}^-}(F_1, \omega_1; \tilde{\omega})$ is an element of $U(C, R)$. Thus, there exists a corresponding transfer function $F_z(s)$ for which $L_z(j\tilde{\omega}) \triangleq G(j\tilde{\omega})F_z(j\tilde{\omega}) \in U(C, R)$. However, this is the contradiction we seek.
(ii) Now, we assume that there exists an unfalsified transfer function $F_2(s)$ such that the curve $L_2(j\omega) \triangleq G(j\omega)F_2(j\omega)$, $\omega \in (-\infty, +\infty)$, does not intersect the open disc $U(C, R)$ but its total number of encirclements of the point $s = -1$ is equal to $i_2 > 0$. By hypothesis,

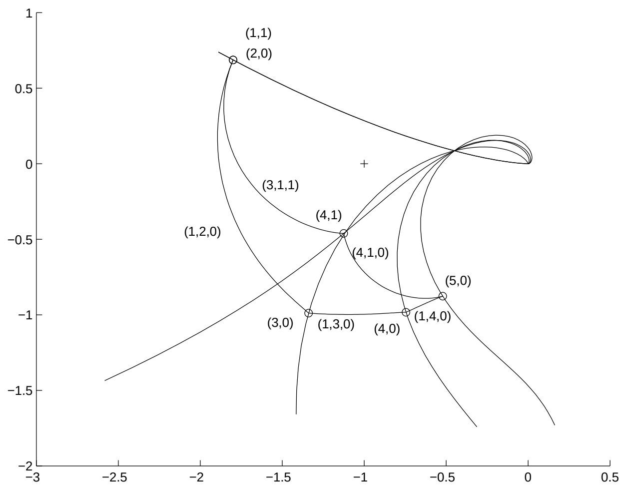

Figure 1. Illustrates the robust instability of the closed loop with the PI controller designed by the ZN frequency response method for the model set $S^5_{\mathbb{R}^-}(e^{-j\pi}, 1)$. The closed curve consisting of the arcs $(1, 2, 0)$, $(1, 3, 0)$, $(1, 4, 0)$, $(4, 1, 0)$ and $(3, 1, 1)$ is the boundary of the set $G_{PI}(j0.5)\cdot \mathcal{F}^5_{\mathbb{R}^-}(e^{-j\pi}, 1; 0.5)$. The corners of this set are denoted by ordered pairs (u, v) according to the appropriate transfer function (7).

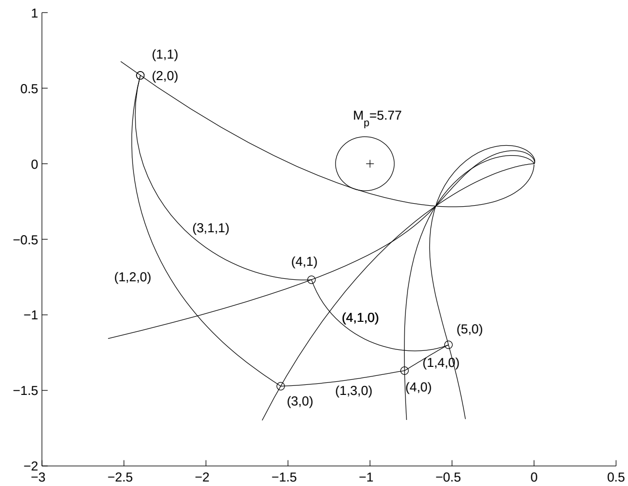

Figure 2. Illustrates the load robust performance of the closed loop with the PID controller designed by the ZN frequency response method for the model set $S^5_{\mathbb{R}^-}(e^{-j\pi}, 1)$. The closed curve consisting of the arcs $(1, 2, 0)$, $(1, 3, 0)$, $(1, 4, 0)$, $(4, 1, 0)$ and $(3, 1, 1)$ is the boundary of the set $G_{PID}(j0.5)\cdot\mathcal{F}^5_{\mathbb{R}^-}(e^{-j\pi}, 1; 0.5)$. The corners of this set are denoted by ordered pairs (u, v) according to the appropriate transfer function (7).

on the other hand, for each ultimate transfer function $F(s)$ the corresponding total number of encirclements is equal to 0. Now, let $F_3(s)$ be an arbitrary ultimate transfer function. Since $S^n_{\mathbb{R}^-}(F_1, \omega_1)$ is pathwise connected (see [9]), there exists a continuous function $[0, 1] \ni \alpha \to F_4(s, \alpha)$ such that $F_4(s, \alpha) \in S^n_{\mathbb{R}^-}(F_1, \omega_1)$ for all $\alpha \in [0, 1]$, $F_4(s, 0) = F_3(s)$ and $F_4(s, 1) = F_2(s)$. If now α sweeps the interval $[0, 1]$, the total number of encirclements of the point $s = -1$ corresponding to the curve $L_4(j\omega, \alpha) \triangleq G(j\omega)F_4(j\omega, \alpha)$ changes its value from 0 to i_2. However, it is only possible if for some $\acute{\alpha} \in [0, 1]$ the curve $L_4(j\omega, \acute{\alpha})$ passes through the point -1. Thus there exists an unfalsified transfer function $F_1(s) \triangleq F_4(s, \acute{\alpha})$ and frequency $\hat{\omega} > 0$ such that $G(j\hat{\omega})F_1(j\hat{\omega}) \in U(C, R)$ and thus we can now proceed as in the case (i) to finish the proof. □

From Lemma 1, it follows that the set of all ultimate transfer functions creates some ”testing set” which may be used for a dramatic reduction in the computational complexity associated with the solution of the robustness problem at hand.

Before a complete parametrization of all the ultimate transfer functions of the family $S^n_{\mathbb{R}^-}(F_1, \omega_1)$ is given, note that without loss of generality the restriction to the case $F_1 = e^{-j\varphi}$, $\varphi \in (0, 2\pi)$, and $\omega_1 = 1$ can be made.

Theorem 1 *(Parameterization of All Ultimate Transfer Functions). The transfer function $F(s)$ is an ultimate transfer function of the family $S^n_{\mathbb{R}^-}(e^{-j\varphi}, 1)$, $\varphi \in [l\frac{\pi}{2}, (l+1)\frac{\pi}{2})$, $l \in \{0, 1, 2, 3\}$, if and only if it can be expressed in the parametric form*

$$F(s) = \frac{K_\nu(\alpha)}{(\tau_\nu(\alpha)s + 1)^{n_1}(\vartheta_\nu(\alpha)s + 1)^{n_2}s^m} \qquad (4)$$

where $\nu = (n_1, n_2, m)$, $n_1 \geq 1$, $n_2 \geq 1$, $m \geq 0$, is an ordered triple of integers (multiindex) ranging over the list

$$\begin{array}{l} (1, l, 0), (1, l + 1, 0), \ldots \\ \ldots, (1, n - 1, 0), (n - 1, 1, 0), \\ (n - 2, 1, 1), (n - 3, 1, 2), \ldots \\ \ldots, (n - l - 1, 1, l) \end{array} \qquad (5)$$

and the parameters $K_\nu(\alpha)$, $\tau_\nu(\alpha)$ and $\vartheta_\nu(\alpha)$ are given by

$$\begin{array}{l} \tau_\nu(\alpha) = \tan\alpha \\ \vartheta_\nu(\alpha) = \tan\frac{\varphi - n_1\alpha - m\frac{\pi}{2}}{n_2} \\ K_\nu(\alpha) = (\tau^2_\nu(\alpha) + 1)^{\frac{n_1}{2}}(\vartheta^2_\nu(\alpha) + 1)^{\frac{n_2}{2}} \end{array} \qquad (6)$$

where α is sweeping the interval

$$I_\nu \triangleq \left[0, \frac{\pi}{2}\right) \bigcap \left(\frac{\varphi - (m + n_2)\frac{\pi}{2}}{n_1}, \frac{\varphi - m\frac{\pi}{2}}{n_1 + n_2}\right].$$

Since the proof of the theorem is rather complicated, it is omitted here but can be found in [9].

Now, some nearly evident consequences of Theorem 1 will be stated. The proofs and further details can also be found in [9].

Let n_1, n_2, m be the fixed integers such that the multiindex $\nu = (n_1, n_2, m)$ belongs to the list (5). The set of all ultimate transfer functions associated with the multiindex ν is parametrized due to Theorem 1 by one parameter $\alpha \in I_\nu$. Thus, its value set for fixed frequency ω is a smooth curve called ν-arc. For each point of this ν-arc there exists just one corresponding ultimate transfer function in the form (4). Observe that the sets of all ultimate transfer functions associated with ν-arcs for different frequencies are the same. Moreover, each endpoint of any ν-arc, where ν belongs to the list (5) corresponds to the transfer function

$$F_{uv}(s) \triangleq \frac{K_{uv}}{(\tau_{uv}s+1)^u s^v} \tag{7}$$

where

$$\tau_{uv} = \tan\frac{\varphi - v\frac{\pi}{2}}{u}, \tag{8}$$

$$K_{uv} = (\tau_{uv}^2+1)^{\frac{u}{2}} \tag{9}$$

and (u, v), $u \geq 1$, $v \geq 0$, is an ordered pair of integers which belongs to the following list

$$\begin{array}{l} (n,0),(n-1,0),\ldots,(l+1,0), \\ (1,l), \\ (n-1,1),(n-2,2),\ldots,(n-l,l). \end{array} \tag{10}$$

An endpoint of a ν-arc is called a (u, v)-node if it corresponds to the transfer function $F_{uv}(s)$ given by (7).

We conclude that the value set $\mathcal{F}^n_{\mathbb{R}^-}(e^{-j\varphi}, 1; \omega)$, $\omega > 0$, $\omega \neq 1$, is bounded by a closed curve which consists of a finite number of ν-arcs from the list (5) and which has corners at the (u, v)-nodes from the list (10).

4 Evaluation of the Ziegler–Nichols Frequency Response Method

In this section, the original ZN frequency response method is tested by means of the set of all ultimate transfer functions of the family $S^n_{\mathbb{R}^-}(e^{-j\varphi}, 1)$, where $\varphi = \pi$ and $n = 5$. Firstly, we observe that each $F(s) \in S^n_{\mathbb{R}^-}(e^{-j\varphi}, 1)$ has the same ultimate gain $K_C = 1$ and the same ultimate period $T_C = 2\pi$. Thus, the method assigns the same controller parameters to all unfalsified transfer functions at hand.

PI Controller: Consider the open loop Nyquist curves $G_{PI}(j\omega)F_{uv}(j\omega))$, where $G_{PI}(s)$ is the PI controller with ZN parameters, $F_{uv}(s)$ is given by (7) and (u, v) belongs to the list (10), where $n = 5$ and $l = 2$ (see Fig. 1). The corners of the value set $\mathcal{F}^5_{\mathbb{R}^-}(e^{-j\pi}, 1; \omega)$ lie on these curves for any $\omega > 0$ as illustrated in Fig. 1. From this fact, it is apparent that unfalsified transfer functions exist which lead to an unstable closed loop. For example, the transfer function

$$F(s) = \frac{32.7}{(0.38s+1)(5.44s+1)^2}$$

has this property.

PID Controller: Similarly, the open loop Nyquist curves $G_{PID}(j\omega)F_{uv}(j\omega)$ for the PID controller $G_{PID}(s)$ with ZN parameters are depicted in Fig. 2. In this case, the closed loop is stable for all unfalsified transfer functions but the largest values of the sensitivity and complementary sensitivity function are very high ($M_p \doteq 5.77$).

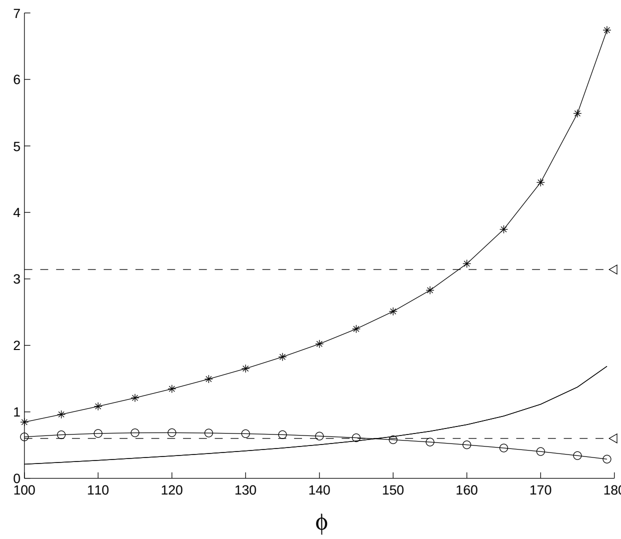

Figure 3. The solution of the ZN experiment based robust design problem with the following data: $F_1 = e^{-j\varphi}$, $\varphi \in (100°, 180°)$, $\omega_1 = 1$, $n = 50$, $T_i/T_d = 4$, $N = 10$, $M_p = 2$. The gain K^* is marked with $\circ$, the integral time T_i^* is marked with $*$ and the derivative time T_d^* is without any mark. The dashed lines correspond to the ZN tuning rule.

5 Numerical solution of the Ziegler–Nichols experiment based robust design problem

In this section, the PID controller design problem defined by Definition 2 is solved by a numerical non-convex optimization procedure based on Lemma 1 and Theorem 1. For brevity, technical details of this procedure are omitted and only the result obtained and some discussion are given.

It is considered that: (i) the unfalsified plant family is specified by $F_1 = e^{-j\varphi}$, $\omega_1 = 1$, $n = 50$, where φ is a variable parameter, (ii) the parameters of the PID controllers are constrained by the following conditions: $T_i/T_d = 4$, $N = 10$, (iii) the disc $U(C, R)$ is specified by (3), where $M_p = 2$. Under these assumptions, the parameters of the optimal PID controller (corresponding to the above mentioned design problem) are approximated by the function in the form

$$f(\varphi) = a_0 e^{a_1\varphi + a_2\varphi^2 + a_3\varphi^3 + a_4\varphi^4} \tag{11}$$

where the independent variable $\varphi \in [90°, 180°]$ is expressed in degrees and coefficients $a_0, \ldots, a_4$ are given in Tab. 1. The dependences of optimal parameters K^* and T_i^* on the parameter φ are depicted in Fig. 3.

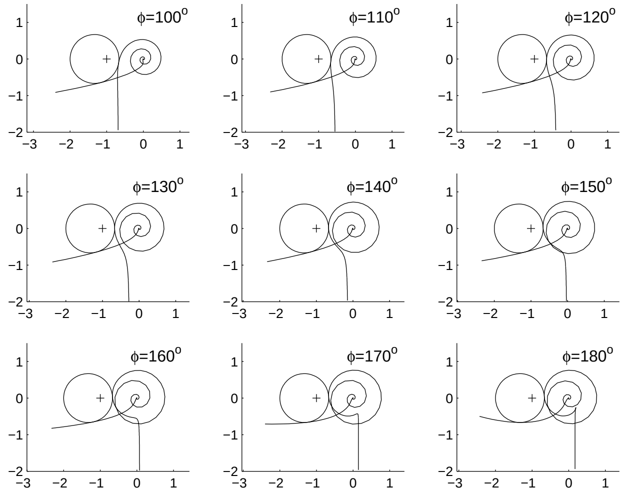

Figure 4. Nyquist curves $G^*(j\omega)F_{11}(j\omega)$ and $G^*(j\omega)F_{n0}(j\omega)$, $n = 50$, for $\varphi = 100°, 110°, \ldots, 180°$.

Table 1. Coefficients of the approximation (11) for the ZN experiment based robust design problem with the following data: $F_1 = e^{-j\varphi}$, $\varphi \in (100°, 180°)$, $\omega_1 = 1$, $n = 50$, $T_i/T_d = 4$, $N = 10$, $M_p = 2$.

	K^*	T_i^*
a_0	3.7463e-10	3.1877e+1
a_1	6.2152e-1	-2.0069e-1
a_2	-6.8294e-3	3.0791e-3
a_3	3.3761e-5	-1.8320e-5
a_4	-6.3897e-8	3.9717e-8

It is interesting that the only two ultimate transfer functions, both in the form (7), play an active role in the circle constraint of open loop Nyquist plots. They correspond with the (u, v) pairs $(1, 1)$ and $(n, 0)$. This observation can be used for a further dramatic reduction in the computational complexity. Open loop Nyquist plots $G^*(j\omega)F_{11}(j\omega)$ and $G^*(j\omega)F_{n0}(j\omega)$, where $G^*(s)$ is the transfer function of the optimal PID controller are depicted in Fig. 4 for several values of parameter φ. Similarly, the step load disturbance responses and step setpoint responses for the same systems are illustrated in Fig. 5 and 6, respectively. By inspection, it may be found that the best shapes of responses are obtained for $\varphi \in (120°, 130°)$. The case $\varphi = 180°$ corresponding to the original ZN method gives a clearly insufficient performance.

6 Conclusions

This paper describes a new nonstandard robust design problem whose formulation has been inspired by the classical ZN frequency response method. It is required to design a fixed PID controller that fulfills the design specifica-

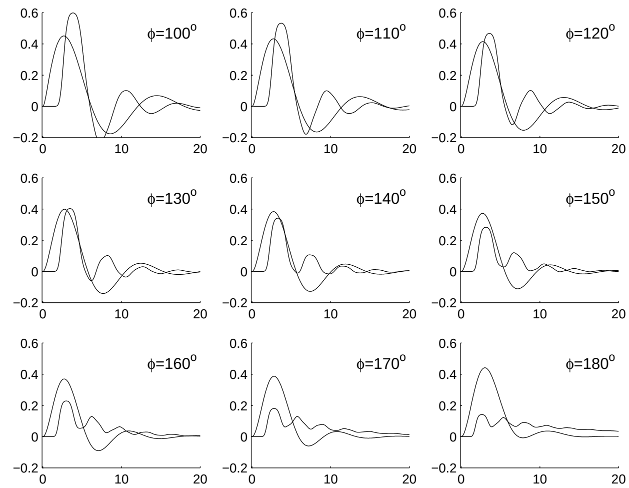

Figure 5. Responses to a load disturbance in the form of a unit step at the plant input of the closed loop systems with the PID controller $G^*(s)$ and plant transfer functions $F_{11}(s)$, $F_{n0}(s)$, $n = 50$, for $\varphi = 100°, 110°, \ldots, 180°$.

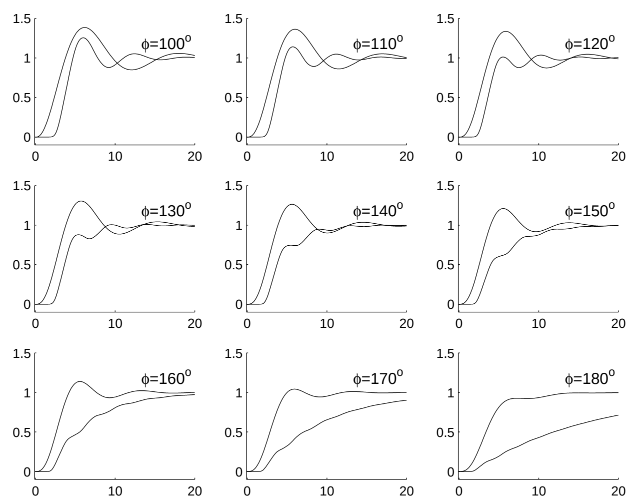

Figure 6. Step responses of the closed loop systems with PID controller $G^*(s)$ and plant transfer functions $F_{11}(s)$, $F_{n0}(s)$, $n = 50$, for $\varphi = 100°, 110°, \ldots, 180°$. The setpoint weighting factor b is fixed at the value 0.65.

tions for all unfalsified systems which are consistent with one sample of the plant frequency response and with some *a priori* information. It is shown that it is sufficient to solve the problem only for some subset of the unfalsified plant family which is significant for the design and which has dimension one. This subset is equal to the set of all ultimate transfer functions. The main result of this paper is the effective parametrization of this subset under mild assumptions. By this fundamental result it is proved that the original ZN frequency response method is unreliable even for very reasonable systems. However, by a clever choice of a point on the Nyquist curve the method can be modified to be surprisingly reliable. The same approach is used in [9] in the case when the experimental data consist of two samples of the plant frequency response. The results obtained have been used in the design of an industrial PID autotuner.

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